7. Determine the amount of money in Helen's account at the end of 3 years if it is compounded:

   a. twice a year.

   b. monthly.

   c. daily.

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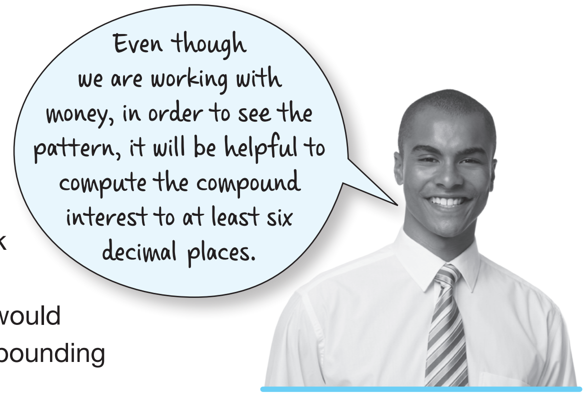
8. What effect does the frequency of compounding have on the amount of money in her savings account?

PROBLEM 3 Easy "e"

Recall that in Problem 2 the variable *n* represented the number of compound periods per year. Let's examine what happens as the interest becomes compounded more frequently.



1. Imagine that Helen finds a different bank that offers her 100% interest. Complete the table to calculate how much Helen would accrue in 1 year for each period of compounding if she starts with $1.

Period of Compounding	$n =$	Formula	Amount
Yearly	1	$1\left(1 + \frac{1}{1}\right)^{1 \cdot 1}$	2.00
Semi-Annually	2	$1\left(1 + \frac{1}{2}\right)^{2 \cdot 1}$	2.25
Quarterly	4	$1\left(1 + \frac{1}{4}\right)^{4 \cdot 1}$	
Monthly	12		
Weekly			
Daily			
Hourly			
Every Minute			
Every Second			

2. Make an observation about the frequency of compounding and the amount that Helen earns. What is it approaching?

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The amount that Helen's earnings approach is actually an irrational number called *e*.

$$e \approx 2.718281828459045 \ldots$$

It is often referred to as the **natural base *e***.

In geometry, you worked with π , an irrational number that was approximated as 3.14159265 . . . and so on. Pi is an incredibly important part of many geometric formulas and occurs so frequently that, rather than write out "3.14159265 . . . " each time, we use the symbol π.

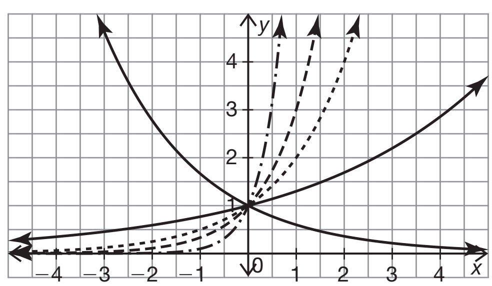
Similarly, the symbol *e* is used to represent the constant 2.718281 . . . It is often used in models of population changes as well as radioactive decay of substances, and it is vital in physics and calculus.

The symbol for the natural base *e* was first used by Swiss mathematician Leonhard Euler in 1727 as part of a research manuscript he wrote at age 21. In fact, he used it so much, *e* became known as Euler's number.

The constant *e* represents continuous growth and has many other mathematical properties that make it unique, which you will study further in calculus.

3. The following graphs are sketched on the coordinate plane shown.
$f(x) = 2^x$, $g(x) = 3^x$, $h(x) = 10^x$, $j(x) = \left(\frac{3}{5}\right)^x$, $k(x) = 1.3^x$.

   a. Label each function.



   b. Consider the function $m(x) = e^x$. Use your knowledge of the approximate value of *e* to sketch its graph. Explain your reasoning.

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c. Using the functions $f(x) = 2^x$, $g(x) = 3^x$, $m(x) = e^x$, approximate the values of $f(2)$, $g(2)$, and $m(2)$ on the number line. Explain your reasoning.

PROBLEM 4 It Keeps Growing and Growing and Growing…

1. The formula for population growth is $N(t) = N_0e^{rt}$. Complete the table to identify the contextual meaning of each quantity.

Quantity	Contextual Meaning
N_0	
r	
t	
$N(t)$	

2. Why is e used as the base?

3. How could this formula be used to represent a decline in population?

4. The population of the city of Fredericksburg, Virginia, was approximately 19,360 in 2000 and has been continuously growing at a rate of 2.9% each year.

   a. Use the formula for population growth to write a function to model this growth.

   b. Use your function model to predict the population of Fredericksburg in 2013.

   c. What value does your function model give for the population of Fredericksburg in the year 1980?

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5. Use a graphing calculator to estimate the number of years it would take Fredericksburg to grow to 40,000 people, assuming that the population trend continues.

Be prepared to share your solutions and methods.

12.3

I Like to Move It

Transformations of Exponential Functions

LEARNING GOALS

In this lesson, you will:

- Dilate, reflect, and translate exponential functions using reference points and transformational function form.
- Investigate graphs of exponential functions through intercepts, asymptotes, intervals of increase and decrease, and end behavior.
- Describe how transformations of exponential functions affect their key characteristics.

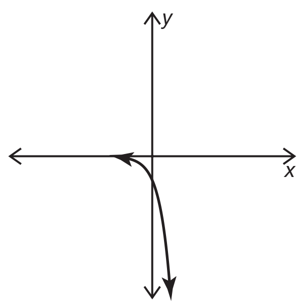
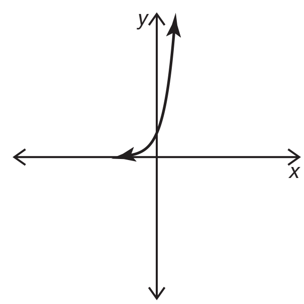
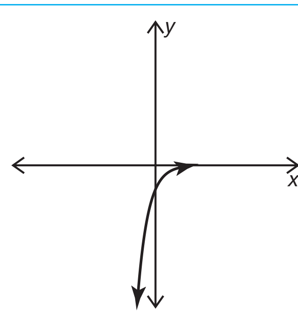
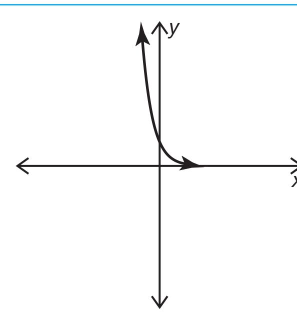
Andy Warhol was an American pop artist whose work explored the relationship between artistic expression, celebrity culture, and advertisement. A recurring theme throughout Warhol's art is the transformation of the mundane and commonplace into art. His most renowned images are silk-screened reproductions of Campbell's soup cans and publicity photographs of pop culture icons like Marilyn Monroe and Elvis Presley.

Have you ever seen any of Andy Warhol's work?

PROBLEM 1 It's the Same . . . But Different!

1. The two tables show four exponential functions and four exponential graphs.
   a. Match the exponential function to its corresponding graph, and write the function under the graph it represents.
   b. Explain the method(s) you used to match the functions with their graphs.

Exponential Functions	
$f(x) = 10^x$	$g(x) = 10^{-x}$
$h(x) = -10^x$	$j(x) = -10^{-x}$

Exponential Graphs	
	
	

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2. Analyze the graphs.

   a. Write an equation for $h(x)$ in terms of $f(x)$. Describe the transformation on $f(x)$.

   b. Write an equation for $g(x)$ in terms of $f(x)$. Describe the transformation on $f(x)$.

   c. Write an equation for $j(x)$ in terms of $f(x)$. Describe the transformation on $f(x)$.

3. Determine the asymptotes, intervals of increase and decrease, and end behavior for each exponential function.

Function	Asymptotes	Intervals of Increase and Decrease	End Behavior
$f(x) = 10^x$			
$g(x) = 10^{-x}$			
$h(x) = -10^x$			
$j(x) = -10^{-x}$			

4. How would the graph of $k(x) = \left(\frac{1}{10}\right)^x$ compare to the graph of $g(x) = 10^{-x}$?

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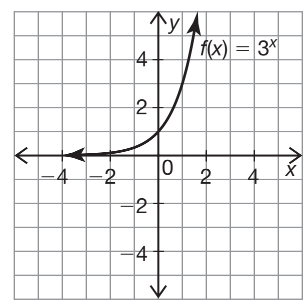
5. How do the transformations on $f(x)$ affect the asymptotes, intervals of increase and decrease, and end behavior?

PROBLEM 2 Keep On Moving

Consider the functions $y = f(x)$ and $g(x) = Af(B(x - C)) + D$. Recall that the D-value translates $f(x)$ vertically, the C-value translates $f(x)$ horizontally, the A-value vertically stretches or compresses $f(x)$, and the B-value horizontally stretches or compresses $f(x)$. Exponential functions are transformed in the same manner.

The function $f(x) = 3^x$ is shown. Recall the key characteristics of basic exponential functions, including a domain of all real numbers, a range of positive numbers, and a horizontal asymptote at $y = 0$.

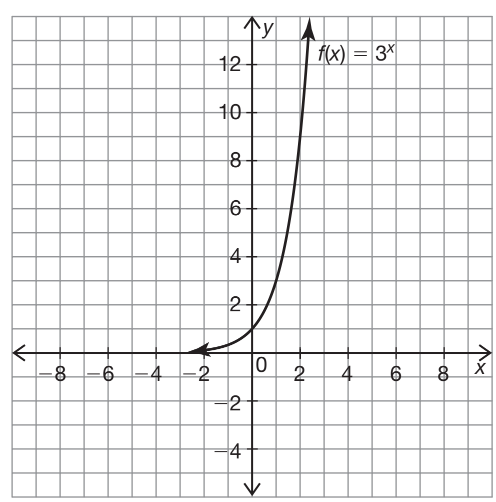


1. Suppose that $a(x) = f(x + 1)$.

   a. Describe the transformation on the graph of $f(x)$ that produces $a(x)$.

   b. Complete the table to determine the corresponding points on $a(x)$, given reference points on $f(x)$. Then, graph and label $a(x)$.

Reference Points on $f(x)$	Corresponding Points on $a(x)$
$\left(-1, \frac{1}{3}\right)$	
$(0, 1)$	
$(1, 3)$	



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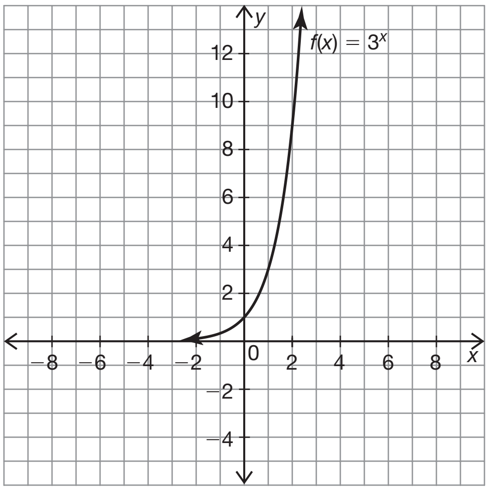
c. Determine the domain, range, and asymptotes of $a(x)$.

2. Suppose that $b(x) = f(x) + 1$.

a. Describe the transformation on the graph of $f(x)$ that produces $b(x)$.

b. Complete the table to determine the corresponding points on $b(x)$, given reference points on $f(x)$. Then, graph and label $b(x)$.

Reference Points on $f(x)$	**Corresponding Points on $b(x)$**
$\left(-1, \frac{1}{3}\right)$	
(0, 1)	
(1, 3)	



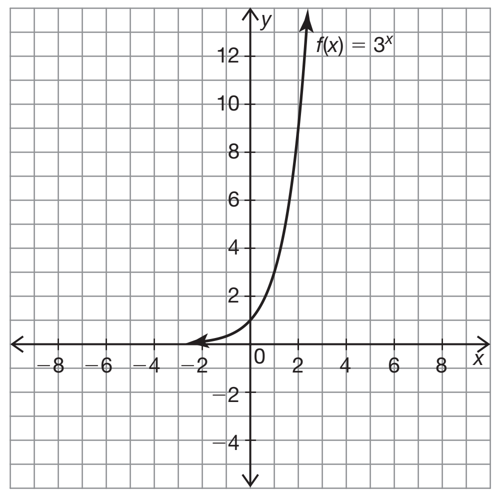
c. Determine the domain, range, and asymptotes of $b(x)$.

3. Suppose that $c(x) = f(x) - 5$.

a. Describe the transformation on the graph of $f(x)$ that produces $c(x)$.

b. Complete the table to determine the corresponding points on $c(x)$, given reference points on $f(x)$. Then, graph and label $c(x)$.

Reference Points on $f(x)$	Corresponding Points on $c(x)$
$\left(-1, \frac{1}{3}\right)$	
(0, 1)	
(1, 3)	



c. Determine the domain, range, and asymptotes of $c(x)$.

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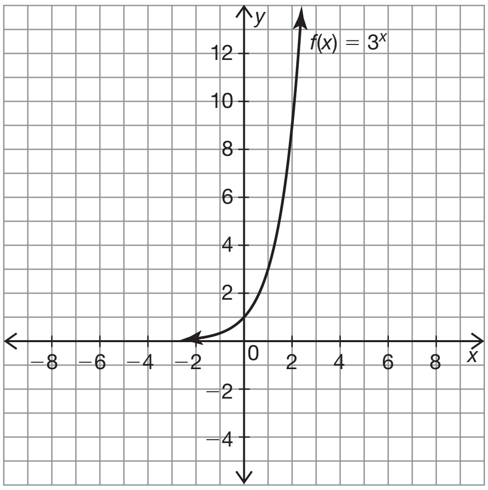
4. Suppose that $d(x) = f(2x)$.

a. Describe the transformation on the graph of $f(x)$ that produces $c(x)$.

b. Complete the table to determine the corresponding points on $d(x)$, given reference points on $f(x)$. Then, graph and label $d(x)$.

Reference Points on $f(x)$	Corresponding Points on $d(x)$
$\left(-1, \frac{1}{3}\right)$	
(0, 1)	
(1, 3)	



c. Determine the domain, range, and asymptotes of $d(x)$.

5. Analyze the transformations performed on $f(x)$ in Questions 1 through 4.

a. Which, if any, of these transformations affected the domain, range, and asymptotes?

b. What generalizations can you make about the effects of transformations on the domain, range, and asymptotes of exponential functions?

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6. Andres and Tomas each described the effects of transforming the graph of $f(x) = 3^x$, such that $p(x) = 3f(x)$.

Andres

$p(x) = 3f(x)$

The A-value is 3 so the graph is stretched vertically by a scale factor of 3.

Tomas

$p(x) = 3f(x)$

$p(x) = 3 \cdot 3^x$

$p(x) = 3^{1+x}$

$p(x) = f(x + 1)$

The C-value is 1 so the graph is horizontally translated 1 unit to the left.

a. Explain Andres' and Thomas' reasoning.

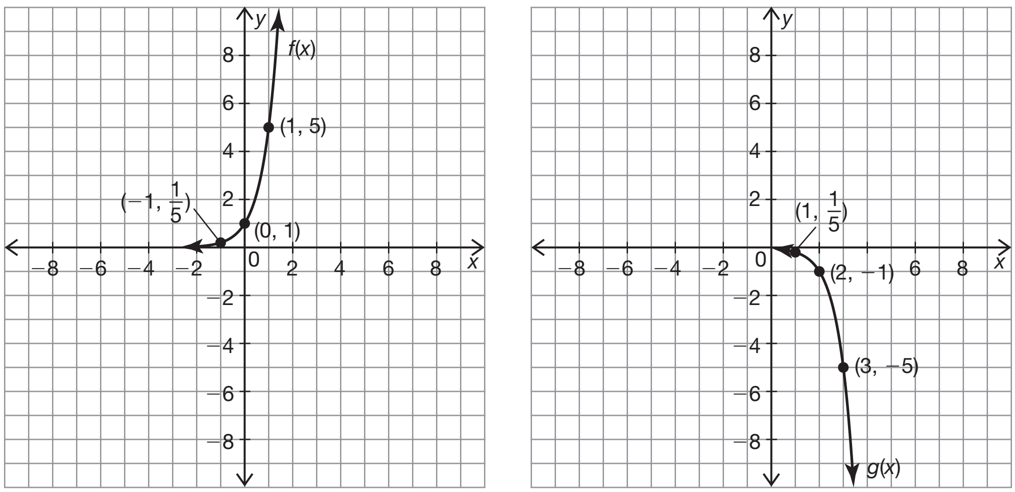
b. Determine the domain, range, and asymptotes of $p(x)$.

c. What generalizations can you make about the effects of vertical dilations on the domain, range, and asymptotes of exponential functions?

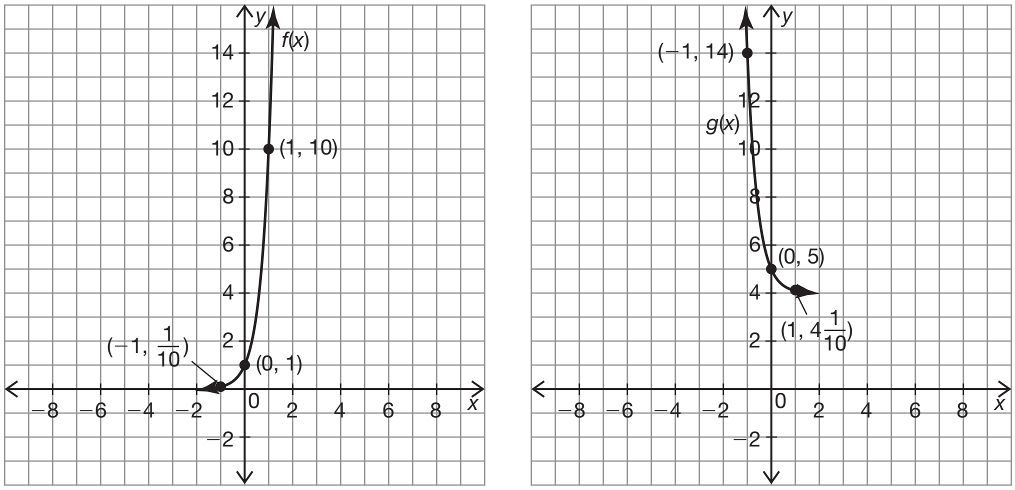
PROBLEM 3 Multiple Transformations

1. Analyze the graphs of $f(x)$ and $g(x)$. Describe the transformations performed on $f(x)$ to create $g(x)$. Then, write an equation for $g(x)$ in terms of $f(x)$. For each set of points shown on $f(x)$, the corresponding points are shown on $g(x)$.

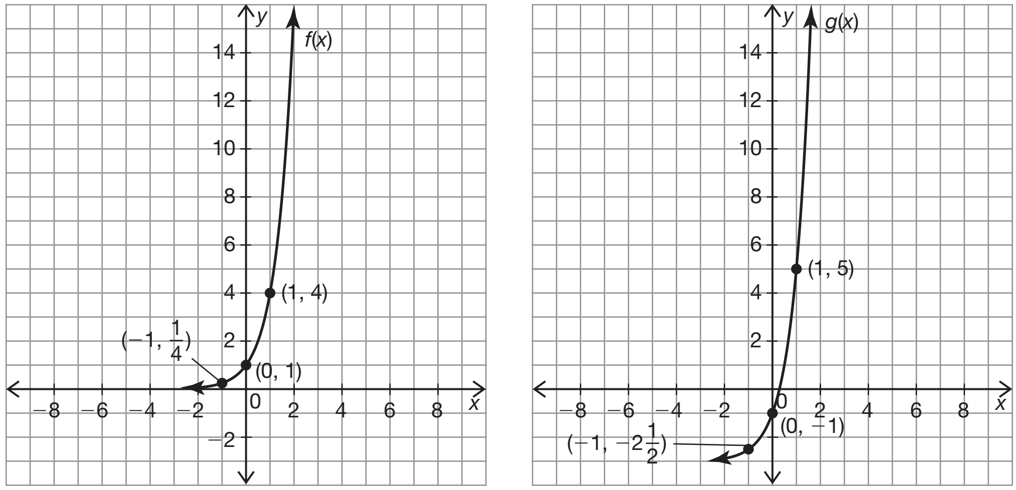
   a. $g(x) =$ ______________________



   b. $g(x) =$ ______________________



c. $g(x) =$ ____________________



2. The equation for an exponential function $m(x)$ is given. The equation for the transformed function $t(x)$ in terms of $m(x)$ is also given. Describe the graphical transformation(s) on $m(x)$ that produce(s) $t(x)$. Then, write an exponential equation for $t(x)$.

a. $m(x) = 2^x$
$t(x) = 0.5m(x + 3)$

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b. $m(x) = e^x$
$t(x) = -m(x) - 1$

c. $m(x) = 6^x$
$t(x) = 2m(-x)$

Be prepared to share your solutions and methods.