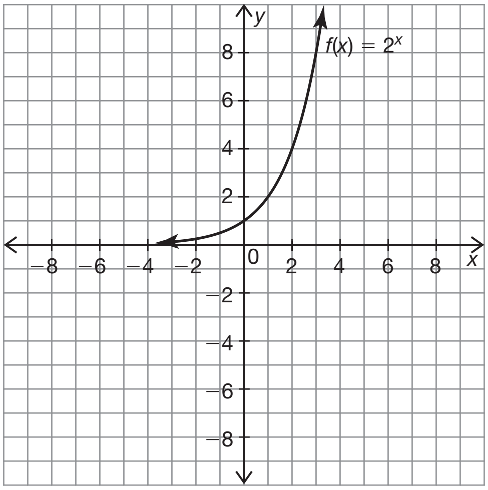
PROBLEM 1 Return of the Inverse

Consider the table and graph for the basic exponential function $f(x) = 2^x$.

x	$f(x) = 2^x$
-3	$\frac{1}{8}$
-2	$\frac{1}{4}$
-1	$\frac{1}{2}$
0	1
1	2
2	4
3	8



You learned that the key characteristics of basic exponential functions are:

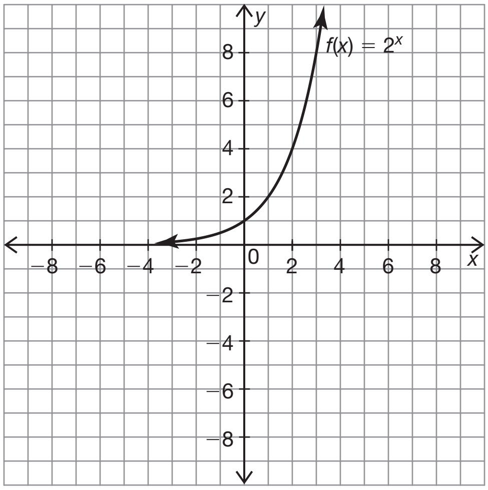
- The domain is the set of all real numbers.
- The range is the set of all positive numbers.
- The y-intercept is (0, 1).
- There is no x-intercept.
- There is a horizontal asymptote at $y = 0$.
- The function increases over the entire domain.
- As x approaches negative infinity, $f(x)$ approaches 0.
- As x approaches positive infinity, $f(x)$ approaches positive infinity.

12

Recall that for any function *f* with ordered pairs (x, y), or (x, $f(x)$), the inverse of the function *f* is the set of all ordered pairs (y, x), or ($f(x)$, x).

1. Graph the inverse of $f(x) = 2^x$ on the same coordinate plane as $f(x)$. Complete the table of values for the inverse of $f(x)$.

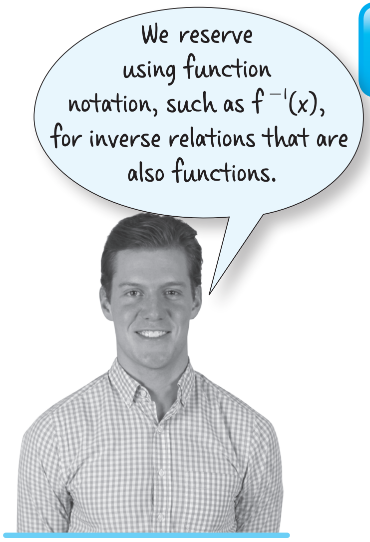
x	***y***



2. Analyze the key characteristics of the inverse of $f(x) = 2^x$.

a. Is the inverse of $f(x) = 2^x$ a function? Explain your reasoning.

b. Identify the domain, range, intercepts, asymptotes, intervals of increase and decrease, and end behavior of $f^{-1}(x)$.



c. What do you notice about the domain and range of the exponential function and its inverse?

d. What do you notice about the asymptotes of the exponential function and its inverse?

e. What do you notice about the intervals of increase and decrease of the exponential function and its inverse?

f. What do you notice about the end behavior of the exponential function and its inverse?

g. Write the equation for the inverse of $y = 2^x$. Explain your reasoning.

It is necessary to define a new function in order to write the equation for the inverse of an exponential function. The **logarithm** of a number for a given base is the exponent to which the base must be raised in order to produce the number. If $y = b^x$, then x is the logarithm and can be written as $x = \log_b(y)$. The value of the base of a logarithm is the same as the base in the exponential expression b^x.

For example, the number 3 is the logarithm to which base 2 must be raised to produce the argument 8. The base is written as the subscript 2. The logarithm, or exponent, is the output 3. The argument of the logarithm is 8.

$$\underset{\text{logarithm, or exponent}}{3} = \log_{\underset{\text{base}}{2}}(\overset{\text{argument}}{8})$$

You can write any exponential equation as a logarithmic equation and vice versa.

Example	Exponential Form	⇔	Logarithmic Form
A	$y = b^x$	⇔	$x = \log_b(y)$
B	$16 = 4^2$	⇔	$2 = \log_4(16)$
C	$1000 = 10^3$	⇔	$3 = \log_{10}(1000)$
D	$32 = 16^{1.25}$	⇔	$1.25 = \log_{16}(32)$
E	$a = b^c$	⇔	$c = \log_b(a)$

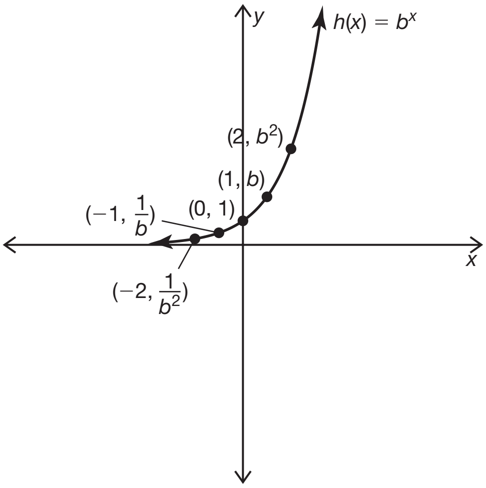
3. Rewrite your equation in Question 2, part (g), in logarithmic form. Label the graph from Question 1 with your equation.

4. Analyze the exponential equation $y = b^x$ and its related logarithmic equation, $x = \log_b(y)$. State the restrictions, if any, on the variables. Explain your reasoning.

<table>
<tr><th colspan="3">$y = b^x \Leftrightarrow x = \log_b(y)$</th></tr>
<tr><th>Variable</th><th>Restrictions</th><th>Explanation</th></tr>
<tr><td>x</td><td></td><td></td></tr>
<tr><td>b</td><td></td><td></td></tr>
<tr><td>y</td><td></td><td></td></tr>
</table>

PROBLEM 2 A Logarithm by Any Other Name . . .

1. The graph of $h(x) = b^x$ is shown. Sketch the graph of the inverse of $h(x)$ on the same coordinate plane. Label coordinates of points on the inverse of $h(x)$.



2. Write the equation for the inverse of $h(x) = b^x$. Label the graph.

3. Do you think all exponential functions are invertible? If so, explain your reasoning. If not, provide a counterexample.

Recall that the logarithm of a number for a given base is the exponent to which the base must be raised in order to produce the number. If $y = b^x$, then the logarithm is written as $x = \log_b y$, where $b > 0, b \neq 1$, and $y > 0$. A **logarithmic function** is a function involving a logarithm.

Logarithms were first conceived by a Swiss clockmaker and amateur mathematician Joost Bürgi but became more widely known and used after the publication of a book by Scottish mathematician John Napier in 1614. Tables of logarithms were originally used to make complex computations in astronomy, surveying, and other sciences easier and more accurate. With the invention of calculators and computers, the use of logarithm tables as a tool for calculation has decreased. However, many real-world situations can be modeled using logarithmic functions.

Two frequently used logarithms are logarithms with base 10 and base *e*. A **common logarithm** is a logarithm with base 10 and is usually written *log* without a base specified.

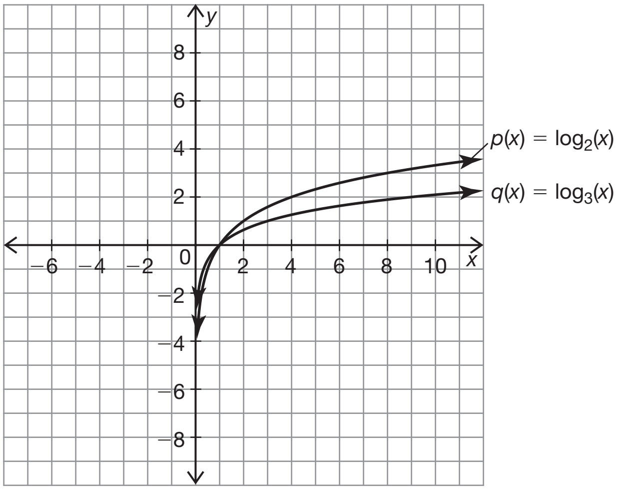
$$c(x) = \log_{10}(x) \quad \Leftrightarrow \quad c(x) = \log x$$

A **natural logarithm** is a logarithm with base *e*, and is usually written as *ln*.

$$n(x) = \log_e(x) \quad \Leftrightarrow \quad n(x) = \ln x$$

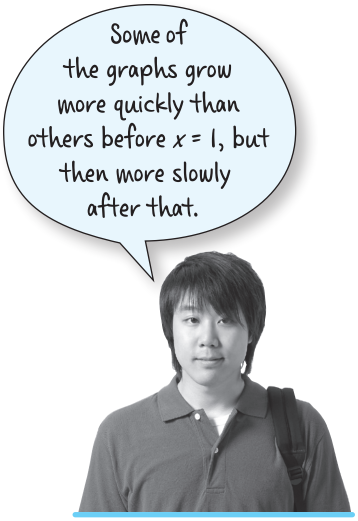
4. The functions $p(x) = \log_2(x)$ and $q(x) = \log_3(x)$ have been graphed for you.

a. Sketch and label the functions $c(x) = \log x$ and $n(x) = \ln x$.



b. Explain how you determined the graphs of $c(x)$ and $q(x)$.

c. Analyze the key characteristics of $p(x)$, $q(x)$, $c(x)$, and $n(x)$. Describe the similarities and differences.



d. What is the inverse of the logarithmic function $c(x) = \log x$?

e. What is the inverse of the logarithmic function $n(x) = \ln x$?