

Answers and explanations are in the boxes below the question.

1) Given the functions  $f(x) = 3x - 4$  and  $g(x) = x^2 - 7$ , find each of the following:

a)  $g(f(4))$

b)  $f(g(x))$

a)  $g(f(4)) = 57$

To do this problem, I will break it into parts. First I will find  $f(4)$ . So I go to the  $f$  function and replace  $x$  with 4.

$$f(4) = 3(4) - 4 = 12 - 4 = 8$$

Now I know that  $f(4) = 8$ , so I can substitute that to get  $g(f(4)) = g(8)$ .

$$g(8) = 8^2 - 7 = 64 - 7 = 57$$

$$\text{So, } g(f(4)) = 57$$

b)  $f(g(x)) = 3x^2 - 25$

To do this problem, I find it easier to substitute "a" instead of "x". I'm going to find  $f(g(a))$  then put  $x$  back in it at the end.

I need to start on the inside, so I'm looking for  $g(a)$  first. That means I will put "a" everywhere there is currently an  $x$  in the  $g$  function.

$$g(a) = a^2 - 7$$

Now, I know that I'm looking for  $f(g(a))$  or  $f(a^2 - 7)$ .

So, in the  $f$  function I will replace  $x$  with  $(a^2 - 7)$ .

$$f(a^2 - 7) = 3(a^2 - 7) - 4 = 3a^2 - 21 - 4 = 3a^2 - 25$$

Since I was originally supposed to find  $f(g(x))$ , I'm going to change my "a" back to an "x". So the final answer is  $f(g(x)) = 3x^2 - 25$

2) Use composition of functions to prove that the following two functions are inverses of each other:

$$f(x) = 2x + 7$$

$$g(x) = \frac{x - 7}{2}$$

To show that two functions are inverses, I must show that they undo each other. That means that when the  $f$  function multiplies everything by 2 then adds 7, I need to make sure that the  $g$  function will "un-multiply" by 2 and "un-add" 7. I must check this in both orders. Again, I find it easier to use a different letter than  $x$  so I'm going to use the letter "a" again.

$$f(g(a)) = f\left(\frac{a-7}{2}\right) = 2\left(\frac{a-7}{2}\right) + 7 =$$

$$\frac{2}{1} \cdot \frac{(a-7)}{2} + 7 =$$

$$\frac{2}{1} \cdot \frac{(a-7)}{2} + 7 =$$

$$a - 7 + 7 = a$$

$$g(f(a)) = g(2a + 7) = \frac{(2a+7)-7}{2} =$$

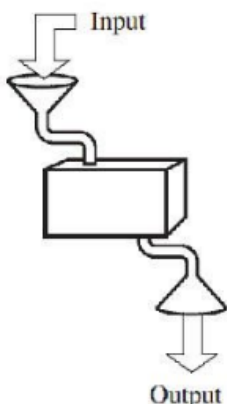
$$\frac{2a+7-7}{2} =$$

$$\frac{2a}{2} = a$$

Since  $f(g(a)) = a$  and  $g(f(a)) = a$ , I know that these functions are inverses.

CP: Additional Practice

It may help you to use a set of function machines to visualize the process until you are confident with composition.



Use the function machines to the left to assist you with the following computations. Show your work.

Given  $f(x) = 9 - x$  and  $g(x) = 2x^2$ , find the following:

1) Find  $f(2)$

2) Find  $g(7)$

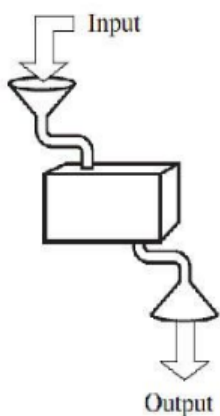
3) Find  $g(f(2))$

4) Find  $f(g(2))$

5) Are the answers to 3 and 4 the same? What does that mean?

6) Find  $f(g(m))$

7) Find  $g(f(m))$



Answers and explanations are in the boxes below the question.

1) Given the functions  $f(x) = 3x - 4$  and  $g(x) = x^2 - 7$ , find each of the following:

a)  $g(f(-2))$

b)  $f(g(w))$

a)  $g(f(-2)) = 93$

To do this problem, I will break it into parts.

First I will find  $f(-2)$ . So I go to the  $f$  function and replace  $x$  with  $(-2)$ .

$$f(-2) = 3(-2) - 4 = -6 - 4 = -10$$

Now I know that  $f(-2) = -10$ , so I can substitute that to get  $g(f(-2)) = g(-10)$ .

$$g(-10) = (-10)^2 - 7 = 100 - 7 = 93$$

$$\text{So, } g(f(-2)) = 93$$

b)  $f(g(w)) = 3w^2 - 25$

I need to start on the inside, so I'm looking for  $g(w)$ .

That means I will put "w" everywhere there is currently an  $x$  in the  $g$  function.  $g(w) = w^2 - 7$

Now I know I'm looking for  $f(g(w))$  or  $f(w^2 - 7)$ .

So in the  $f$  function, I will replace  $x$  with  $(w^2 - 7)$ .

$$f(w^2 - 7) = 3(w^2 - 7) - 4 = 3w^2 - 21 - 4 = 3w^2 - 25$$

So the final answer is  $f(g(w)) = 3w^2 - 25$

2) Use composition of functions to prove that the following two functions are inverses of each other:

$$f(x) = \frac{3x - 5}{2}$$

$$g(x) = \frac{2}{3}x + \frac{5}{3}$$

To show that two functions are inverses, I must show that they undo each other. That means that when the  $f$  function multiplies everything by  $2/3$  then adds  $5/3$ , I need to make sure that the  $g$  function will "un-multiply" by  $2/3$  and "un-add"  $5/3$ . I must check this in both orders. I find it easier to use a different letter than  $x$ , so I'm going to use the letter "a" instead...

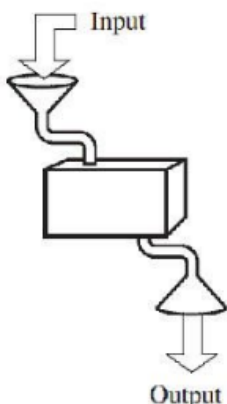
$$f(g(a)) = f\left(\frac{2}{3}a + \frac{5}{3}\right) = \frac{3\left(\frac{2}{3}a + \frac{5}{3}\right) - 5}{2} = \frac{3\left(\frac{2}{3}a\right) + 3\left(\frac{5}{3}\right) - 5}{2} = \frac{2a + 5 - 5}{2} = \frac{2a}{2} = a$$

$$g(f(a)) = g\left(\frac{3a - 5}{2}\right) = \frac{2}{3}\left(\frac{3a - 5}{2}\right) + \frac{5}{3} = \frac{3a - 5}{3} + \frac{5}{3} = \frac{3a}{3} - \frac{5}{3} + \frac{5}{3} = \frac{3a}{3} = a$$

Since  $f(g(a)) = a$  and  $g(f(a)) = a$ , I know that these functions are inverses.

HN: Additional Practice

It may help you to use a set of function machines to visualize the process until you are confident with composition.



Use the function machines to the left to assist you with the following computations. Show your work.

Given  $f(x) = 9 - x^2$  and  $g(x) = \frac{3}{5}x + 8$ , find the following:

1) Find  $f(2)$

2) Find  $g(5)$

3) Find  $g(f(2))$

4) Find  $f(g(2))$

5) Are the answers to 3 and 4 the same? What does that mean?

6) Find  $f(g(m))$

7) Find  $g(f(m))$

