Progress Check Follow-Up: Solving Exponential Equations (B13-B15) Name: ____________________________

Solve for x and show your work below. Round to the nearest hundredth!

1. $130 = x^3 + 5$

To solve this equation for x, we need to get x by itself. Since x is the base, we should be able to solve using "normal" rules of solving an equation (backwards PEMDAS). First, we would need to undo the +5 and move it to the left side of the equation. We would do this by subtracting 5 from both sides.

$$\begin{aligned} 130 &= x^3 + 5 \\ -5 &\quad\; -5 \\ 125 &= x^3 \end{aligned}$$

Now the only thing left to do is undo the power of 3 that is on the x. To do that we need to take a cube root of both sides.

$$\sqrt[3]{125} = \sqrt[3]{x^3}$$
$$5 = x$$

So, the solution to this equation is $x = 5$.

2. $3^x = 27$

To solve this equation, we need to get x by itself. But since x is the exponent in this equation, the only way to solve for it is to use a log. First, we need to check and make sure that the $\boldsymbol{base^{exponent}}$ term is isolated. Since there are no other numbers on that side of the equation that might cause a vertical transformation (terms being added, subtracted, or multiplied outside or around the $\boldsymbol{base^{exponent}}$ term), then we know that it is isolated already and ready to convert to the log form. To turn it into the log form, we can follow the conversion rule below:

$$base^{exponent} = argument \;\rightarrow\; \log_{base} argument = exponent$$

In this problem the base is 3, the exponent is x, and the argument is 27.

So the new converted log form would be $\log_3 27 = x$.

Now, we have x by itself, but we still have to evaluate the log to get a simplified answer.

To evaluate the log, we will use the change of base formula:

$$\log_3 27 = \frac{\log 27}{\log 3} = \frac{\log(argument)}{\log(base)}$$

The two new logs are both common logs (understood base of 10), so we can put them in the calculator using the common log button and evaluate to find x:

$$x = \frac{\log 27}{\log 3} = 3$$

So that means that our solution is $x = 3$.

3. Taniya has $2350 to invest in an account that earns 1.2% interest compounded quarterly. How many years will she need to keep her money invested in order to have $2500? Write an exponential equation, then solve algebraically. If you get stuck, try solving graphically.

Since this problem says "compounded quarterly", we know that we will need to use the compound interest formula $A = A_0\left(1 + \frac{r}{n}\right)^{nt}$, where A_0 = initial amount invested = \$2350, r = interest rate as a decimal = 0.012, n = number of times compounded per year = quarterly = 4, t = time (unknown), and A = ending amount after t years = \$2500.

So, plugging that information into the compound interest formula, we would get: $2500 = 2350\left(1 + \frac{0.012}{4}\right)^{4t}$

Since our time, t, is unknown and it is part of our exponent, then we know we will need to turn our original exponential form into the log form. But first, we must isolate our $\boldsymbol{base^{exponent}}$ term as explained in #2 earlier. Since we have a number in front of our $\boldsymbol{base^{exponent}}$ term being multiplied, we would need to undo it by dividing both sides of the equation by 2350:

$$\frac{2500}{2350} = \frac{2350\left(1 + \frac{0.012}{4}\right)^{4t}}{2350} \quad \rightarrow \quad 1.063829787 = \left(1 + \frac{0.012}{4}\right)^{4t}$$

We can also use the calculator to simplify our base inside the parentheses to make it easier to work with:

$$1.063829787 = (1.003)^{4t}$$

Now that we have our $\boldsymbol{base^{exponent}}$ term isolated on the right side of the equation we can convert it into the log form as explained in #2 earlier: $\log_{1.003} 1.063829787 = 4t$. And using the change of base formula, we can turn that into: $\frac{\log 1.063829787}{\log 1.003} = 4t$. We can plug the left side into the calculator and simplify to get $20.65605676 = 4t$. Then we can easily solve for t (get t by itself) by dividing both sides by 4: $\frac{20.65605676}{4} = \frac{4t}{4}$

And so we will end up with the solution $5.164014189 = t$ which is approximately $t \approx 5.16\ years$.

Additional Practice:

Solve for x and show your work for each problem. Round to the nearest hundredth if necessary.

1. $1060 = x^2 - 4$

2. $8^x + 215 = 727$

3. Annabelle is going to deposit $500 in a savings account that earns interest at a rate of 7.2% compounded semiannually. How long will Annabelle need to keep her money in the savings account to get an account balance of $1500?