EXPONENTIALS & LOGARITHMS TEST REVIEW p. 1 **Name:** Key #______

Below are many of our learning targets from Unit B. Beside each target you will see a reference to the assignment that corresponds to it. These assignments, your progress checks, and your quizzes can all be reviewed to help you better prepare for the unit test. Some additional practice has also been included to help you review so that you may do your best on the unit test.

- I can list values of an inverse given a table, graph, or equation of a function that has an inverse. (B1, B2)
- I can recognize the inverse of a given relation from a table, graph, or equation. (B2)

1. Given that (-6, 0) is a point on the graph of *f(x)*, what is the corresponding point on the graph of $f^{-1}(x)$?

switch x and y

- I can write the inverse of a function in standard notation by replacing the x in my equation with y. (B3)

2. Find the inverse of $f(x) = 2x + 5$.

$y = 2x + 5$
$x = 2y + 5$
$-5 \quad -5$
$x - 5 = 2y$

$y = \frac{x-5}{2}$ or $y = \frac{1}{2}x - \frac{5}{2}$
$f^{-1}(x) = \frac{1}{2}x - \frac{5}{2}$

3. Find the inverse of $g(x) = 3^{x+2}$.

$y = 3^{x+2}$
$x = 3^{y+2}$

base = 3
exp = y + 2
arg = x

$\log_3 x = y + 2$
$\log_3 x - 2 = y$

$g^{-1}(x) = \log_3 x - 2$

- I can use function notation. (B4)
- I can find the composition of two functions with integer and variable inputs. (B5)

4. Consider the functions $f(x) = -2x + 3$ and $g(x) = 5 - x^2$. CP: Evaluate $f(g(c))$. Show your work. HN: Evaluate $g(f(c))$. Show your work.

CP: $f(g(c)) = f(5 - \cancel{x^2}c^2)$
$= -2(5 - c^2) + 3$
$= -10 + 2c^2 + 3$
$f(g(c)) = 2c^2 - 7$

HN: $g(f(c)) = g(-2c + 3)$
$= 5 - (-2c + 3)^2$
$= 5 - (-2c + 3)(-2c + 3)$
$= 5 - (4c^2 - 12c + 9)$
$g(f(c)) = -4c^2 + 12c - 4$

- I can use composition of functions to verify that *g(x)* and *f(x)* are inverses by showing that *f(g(x)) = g(f(x)) = x*. (B6)

5. Use composition of functions to determine whether the given functions are inverses of each other.

CP: $f(x) = 10x^{\frac{1}{5}}$ $\qquad g(x) = \left(\frac{x}{10}\right)^5$

HN: $f(x) = \sqrt[5]{(2x + 4)} - 3$ $\qquad g(x) = \frac{1}{2}(x + 3)^5 - 2$

CP: $f(g(a)) = f\left[\left(\frac{a}{10}\right)^5\right]$
$= 10\left[\left(\frac{a}{10}\right)^5\right]^{\frac{1}{5}}$
$= 10\left(\frac{a}{10}\right)$
$= a$

$g(f(a)) = g(10a^{\frac{1}{5}})$
$= \left(\frac{10a^{\frac{1}{5}}}{10}\right)^5$
$= (a^{\frac{1}{5}})^5$
$= a$

HN: $f(g(a)) = f\left[\frac{1}{2}(a+3)^5 - 2\right]$
$= \sqrt[5]{2\left(\frac{1}{2}(a+3)^5 - 2\right) + 4} - 3$
$= \sqrt[5]{(a+3)^5 - 4 + 4} - 3$
$= \sqrt[5]{(a+3)^5} - 3$
$= (a+3) - 3$
$= a$

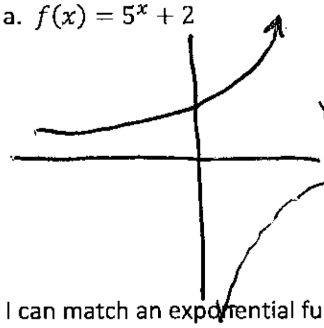
$g(f(a)) = g\left(\sqrt[5]{2a+4} - 3\right)$
$= \frac{1}{2}\left[\sqrt[5]{2a+4} - 3 + 3\right]^5 - 2$
$= \frac{1}{2}\left[\sqrt[5]{2a+4}\right]^5 - 2$
$= \frac{1}{2}(2a+4) - 2$
$= a + 2 - 2 = a$

Key p. 2

- I can determine if a function is one-to-one by graphing its inverse. (B7)

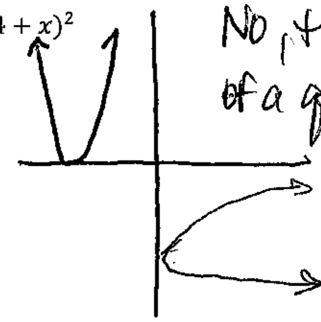
6. Sketch the following functions. Tell whether or not they are one-to-one and explain how you know.

a. $f(x) = 5^x + 2$



Yes b/c exponential and log are both functions

b. $g(x) = (4+x)^2$



No, the inverse of a quadratic is not a function

- I can match an exponential function to its equation, graph, and table. (B9, B10)
- I can recognize the characteristics of exponential growth and decay. (B9, B10)

7. Determine whether each function represents exponential growth or decay.

a. $f(x) = 8^x$

growth
$b > 1$

b. $g(x) = \left(\frac{1}{8}\right)^{-x}$

growth because
$\left(\frac{1}{8}\right)^{-x} = \left(\frac{8}{1}\right)^{x}$

c. $h(x) = \left(\frac{2}{7}\right)^{x}$

decay
$0 < b < 1$

- I can understand why we use base e for compounding continuously. I can use the compounded continuously formula to find time, amount and interest rates. (B10, B11)

8. Six years ago, Dimitri invested \$5000 in an account in which the interest is compounded continuously at an annual interest rate of 5.2%. Determine the current amount of money in the account to the nearest cent.

$$A(t) = Pe^{rt}$$
$$A(6) = 5000 \cdot e^{.052(6)}$$
$$= 5000 \cdot e^{.312}$$
$$= \$6830.77$$

9. Five years ago, Vaughn invested money into an account in which the interest is compounded continuously at an annual interest rate of 4%. The account is currently valued at \$1099.26. Determine the amount of money Vaughn invested to the nearest dollar.

$$A(t) = Pe^{rt}$$
$$1099.26 = P \cdot e^{.04(5)}$$
$$1099.26 = P \cdot e^{0.2}$$
$$\frac{1099.26}{e^{0.2}} = P$$
$$\$900 = P$$

- I can understand the pieces to a half-life equation. (B8)
- I can write exponential equations from a context including compound interest, half-life, and population change (B11)

10. An alien radioactive isotope has a half-life of 238 years. If you start with a sample of 8 kg, how much will be left in 100 years?

$$h(t) = a_0\left(\frac{1}{2}\right)^{\frac{t}{P}} \quad h(100) = 8\left(\frac{1}{2}\right)^{\frac{100}{238}} = 5.9787 \text{ Kg}$$

11. If you deposit \$4000 into an account paying 4% annual interest compounded quarterly, how much money will be in the account after 5 years?

$$A(t) = P\left(1 + \frac{r}{n}\right)^{n \cdot t}$$
$$A(5) = 4000\left(1 + \frac{.04}{4}\right)^{4 \cdot 5}$$
$$= 4000(1.01)^{20} = \$4880.76$$

Key p.3

- I can describe key features of an exponential function that has been translated including asymptotes, domain, range, end behavior, and intervals of increase/decrease. (B11, B12)

12. Identify the characteristics of the graph of the function (

CP: $k(x) = \left(\frac{1}{3}\right)^x + 4$

HN: $j(x) = \ln(x-1) + 3$

a. Domain: CP: $\mathbb{R}$ or $(-\infty,\infty)$
HN: $x > 1$ or $(1,\infty)$

b. Range: CP: $y > 0$ or $(0,\infty)$
HN: $\mathbb{R}$ or $(-\infty,\infty)$

c. Asymptotes:
CP: $y = 4$
HN: $x = 1$

d. Intercepts:
CP: $(0,5)$ no x-intercept
HN: $(1.0499, 0)$ no y-intercept

e. End behavior: CP: As $x \to \infty$, $y \to 4$
As $x \to -\infty$, $y \to \infty$
HN: As $x \to \infty$, $y \to \infty$
~~As $x \to -\infty$~~ As $x \to 1$, $y \to -\infty$

f. Intervals of increase or decrease:
CP: Decreases $(-\infty,\infty)$
HN: Increases $(1,\infty)$

- I can apply the rules of translation and stretch/compression to any function. (B12)

13. The equation for a polynomial function *p(x)* is given. The equation for the transformed function *t(x)* in terms of *p(x)* is also given. Describe the transformation(s) performed on *p(x)* that produced *t(x)*. Then, write an equation for *t(x)* in terms of x.

$p(x) = 3^x$ $\quad$ $t(x) = -p(x-2) + 4$

$t(x) = -3^{(x-2)} + 4$

The negative out front reflects over x-axis.
The $(x-2)$ indicates a shift 2 units to the right
The +4 shifts the graph up 4 units

- I understand the purpose of a log function. (B13)

14. Evaluate each expression.

a. $\log_5 125 =$ 3
because $5^3 = 125$
or $\frac{\log 125}{\log 5} = 3$

b. $\log_3 \frac{1}{243} =$ -5 because $3^{-5} = \frac{1}{243}$

$3^x = \frac{1}{243}$ $\longrightarrow$ $\frac{\log\left(\frac{1}{243}\right)}{\log(3)}$

$\frac{1}{3^x} = \frac{1}{243}$

$\frac{1}{3^x} = \frac{1}{3^5} \rightarrow 3^{-x} = \frac{1}{3^5}$

$x = -5$

- I can transition from exponential form to log form. (B14)

15. Arrange the given terms to create a true exponential equation and a true logarithmic equation: 2, 256, 8.

Exponential Equation: $2^8 = 256$

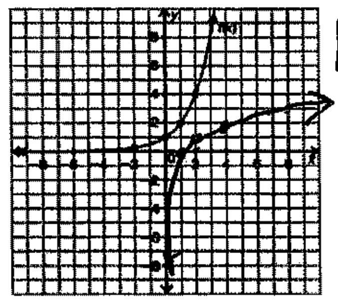
Logarithmic Equation: $\log_2 256 = 8$

Key p.4

- I can find the inverse of an exponential function. I can define a log function. (B13)
- I can compare key features of exponential and log graphs. (B14)

16. The graph of the function *f(x)* is shown.
a. Draw a graph of the inverse function, $f^{-1}(x)$, on the same coordinate grid. Show the three corresponding reference points on your graph.

$f^{-1}(x)$ (1,0)
(2,1)
(4,2)



Key points
(0,1)
(1,2)
(2,4)
$f(x) = 2^x$

$y = 2^x$
$x = 2^y$
$\log_2 x = y$
$f^{-1}(x) = \log_2 x$

b. If $f(x) = 2^x$, write the equation for the function $f^{-1}(x) = \log_2 x$

c. What is the asymptote for *f(x)*? $y = 0$

What is the asymptote for $f^{-1}(x)$? $x = 0$

- I can solve an exponential equation by converting to log form and solving using change of base formula. (B15, B16, B18)

17. Solve the following equations. Round answers to the nearest thousandth.

a. $3^{5x} + 4 = 120$
$-4 \quad -4$
$3^{5x} = 116$
base = 3
exp = 5x
arg = 116

$\log_3 116 = 5x$
$\frac{\log 116}{\log 3} = 5x$
$4.3269 = 5x$
$0.8654 = x$

b. $6^{x+2} = 75$
base = 6
exp = x+2
arg = 75

$\log_6 75 = x+2$
$\frac{\log 75}{\log 6} = x+2$
$2.4096 = x+2$
$0.4096 = x$

- I can solve exponential word problems using logs. (B16, B18)

18. If you deposit \$5000 into an account paying 6% annual interest compounded monthly, how long until there is \$8000 in the account?

12

$A(t) = P\left(1 + \frac{r}{n}\right)^{nt}$
$8000 = 5000\left(1 + \frac{0.06}{12}\right)^{12t}$
$\frac{8000}{5000} = \frac{5000(1.005)^{12t}}{5000}$

$1.6 = (1.005)^{12t}$
$\log_{1.005} 1.6 = 12t$
$\frac{\log 1.6}{\log 1.005} = 12t$
$94.2355 = 12t$
$7.8530 = t$ (Almost 8 years)

base = 1.005
exp = 12t
arg = 1.6

- I can solve log equations. (B17, B18)

19. Solve the following equations.

a. CP: $\log_{16} 32 = n$
a. HN: $\log(4m - 2) = 3$

CP: $\frac{\log 32}{\log 16} = n$
$2 = n$

HN: $10^3 = 4m - 2$
$1000 = 4m - 2$
$1002 = 4m$
$250.5 = m$

b. CP: $\log_w 1728 = 3$
b. HN: $\log_{(5m+4)} 65536 = 4$

CP: $w^3 = 1728$
$\sqrt[3]{w^3} = \sqrt[3]{1728}$
$w = 12$

HN: $(5m+4)^4 = 65536$
$\sqrt[4]{(5m+4)^4} = \sqrt[4]{65536}$
$5m + 4 = 16$
$5m = 12$
$m = 2.4$