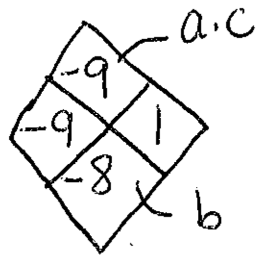
Station 1: I can factor a trinomial where a≠1.

a) Factor $3y^2-8y-3$

$a=3$
$b=-8$
$c=-3$



$3y^2-9y+1y-3$

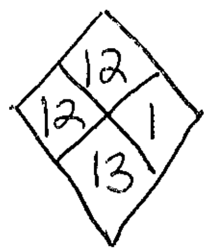
$(3y^2-9y)+(1y-3)$

$3y(y-3)+1(y-3)$

$(y-3)(3y+1)$

b) Factor $4m^2+13m+3$

$a=4$
$b=13$
$c=3$



$4m^2+12m+1m+3$

$(4m^2+12m)+(1m+3)$

$4m(m+3)+1(m+3)$

$(m+3)(4m+1)$

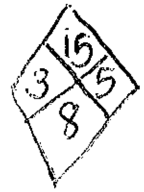
Station 1: I can factor a trinomial where a = 1.

Factor each quadratic expression.

a. $x^2 + 8x + 15 =$ $(x+3)(x+5)$

$x^2 - 8x + 15 =$ $(x-3)(x-5)$

a=1
b=8
c=15



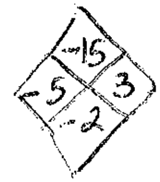
a=1
b=-8
c=15

$x^2 + 2x - 15 =$ $(x-3)(x+5)$

$x^2 - 2x - 15 =$ $(x-5)(x+3)$

a=1
b=2
c=-15

a=1
b=-2
c=-15



b. $x^2 + 10x + 24 =$ $(x+6)(x+4)$

$x^2 - 10x + 24 =$ $(x-6)(x-4)$

a=1
b=10
c=24

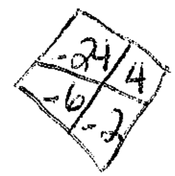
a=1
b=-10
c=24

$x^2 + 2x - 24 =$ $(x-4)(x+6)$

$x^2 - 2x - 24 =$ $(x-6)(x-4)$

a=1
b=2
c=-24

a=1
b=-2
c=-24



Station 2: I can factor the difference of two squares.

First multiply these factors:

1) (x-4)(x+4) $x(x+4) - 4(x+4) = x^2 + 4x - 4x - 16 = x^2 - 16$

$x^2 \quad 4^2$

2) (x+3)(x-3) $x(x-3) + 3(x-3) = x^2 - 3x + 3x - 9 = x^2 - 9$

$x^2 \quad 3^2$

3) (3x-1)(3x+1) $3x(3x+1) + -1(3x+1) = 9x^2 + 3x - 3x - 1 = 9x^2 - 1$

$(3x)^2 \quad 1^2$

4) $(2x^2+3)(2x^2-3)$ $2x^2(2x^2-3) + 3(2x^2-3) = 4x^4 - 6x^2 + 6x^2 - 9 = 4x^4 - 9$

$(2x^2)^2 \quad 3^2$

What pattern do you notice is always true when you multiply (a+b)(a-b)?

The middle terms always add to zero. The first is a square and the last is a square.

Now use the pattern in reverse:

5) Factor $9x^2$-25

6) Factor 16-z^4

5) $9x^2 - 25 = (3x+5)(3x-5)$

$(3x)^2 \quad (5)^2$

6) $16 - z^4 = (4 - z^2)(4 + z^2)$

$(4)^2 \quad (z^2)^2 \quad (2)^2 \quad (z)^2$

$= (2+z)(2-z)(4+z^2)$

Station 3: I can solve a quadratic equation by factoring.

The solutions to a quadratic equation are called *roots*. The **roots** indicate where the graph of a quadratic equation crosses the *x*-axis. So, roots, zeros, and *x*-intercepts are all related.

To calculate the roots of a quadratic equation using factoring:

- Perform transformations so that one side of the equation is equal to zero.
- Factor the quadratic expression on the other side of the equation.
- Set each factor equal to zero.
- Solve the resulting equations for the roots. Check each solution in the original equation.

You can calculate the roots for the quadratic equation $x^2 - 4x = -3$.

$x^2 - 4x = -3$

$x^2 - 4x + 3 = -3 + 3$

$x^2 - 4x + 3 = 0$

$(x - 3)(x - 1) = 0$

$(x - 3) = 0$ or $(x - 1) = 0$

$x - 3 + 3 = 0 + 3$ or $x - 1 + 1 = 0 + 1$

$x = 3$ or $x = 1$

Check:

$x^2 - 4x = (3)^2 - 4(3) = 9 - 12 = -3$

$x^2 - 4x = (1)^2 - 4(1) = 1 - 4 = -3$

Always set it equal to 0.
Use the diamond to factor.

Solve by factoring:

a) $3x^2-22x+7=0$

b) $w^2 + 8w = -7$

a) $3x^2 - 22x + 7 = 0$ $a = 3$, $b = -22$, $c = 7$

$3x^2 - 21x + -1x + 7 = 0$

$(3x^2 - 21x) + (-1x + 7) = 0$

$3x(x-7) + -1(x-7) = 0$

$(x-7)(3x-1) = 0$

$x - 7 = 0$ or $3x - 1 = 0$

$x = 7$ $3x = 1$

$x = \frac{1}{3}$

b) $w^2 + 8w = -7$ $a = 1$, $b = 8$, $c = 7$

$w^2 + 8w + 7 = 0$

$w^2 + 7w + 1w + 7 = 0$

$(w^2 + 7w) + (1w + 7) = 0$

$w(w+7) + 1(w+7) = 0$

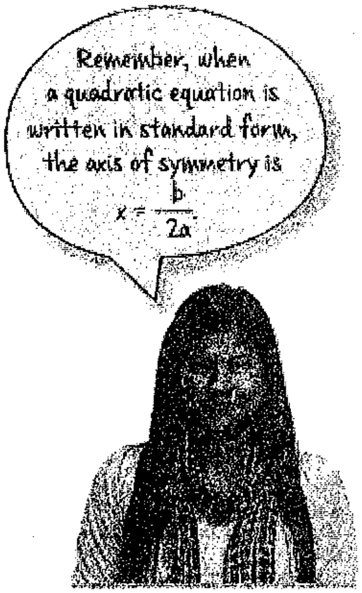
$(w+7)(w+1) = 0$

$w + 7 = 0$ or $w + 1 = 0$

$w = -7$ $w = -1$

Station 4: I can identify the axis of symmetry without a calculator.

Standard Form of a Quadratic Function: $y = ax^2 + bx + c$



Given each quadratic equation, write the equation for the axis of symmetry of the graph.

a) $f(x) = 6x^2 + 24x - 10$

b) $h(t) = -16t^2 + 120t + 3$

A) $a = 6$, $b = 24$, $c = -10$

a.o.s.

$X = -\frac{b}{2a}$

$X = \frac{-24}{2(6)}$

$X = \frac{-24}{12}$

$\boxed{X = -2}$

B) $a = -16$, $b = 120$, $c = 3$

a.o.s.

$X = -\frac{b}{2a}$

$X = \frac{-120}{2(-16)}$

$X = \frac{-120}{-32}$

$\boxed{X = 3.75}$

Station 4: I can sketch a graph of a quadratic without my calculator.

1. A group of students are working together on the problem shown.

   Write a quadratic function in factored form to represent a parabola that opens downward and has zeros at (4, 0) and (−1, 0).

   Maureen
   My function is
   $h(x) = -(x - 4)(x + 1)$.

   Micheal
   My function is
   $d(x) = \frac{1}{2}(x - 4)(x + 1)$.

   Tim
   My function is
   $m(x) = 2(x - 4)(x + 1)$.

   Tom
   My function is
   $j(x) = -2(x - 4)(x + 1)$.

   Dianne
   My function is
   $t(x) = -0.5(x - 4)(x + 1)$.

   Judy
   My function is
   $f(x) = -(x + 4)(x - 1)$.

   a. Use your graphing calculator to graph each student's function. What are the similarities among all the graphs? What are the differences among the graphs.

   all open downward except Tim's and Michael's
   all have zeros at (4,0) and −1,0) except Judy.

   b. How is it possible to have more than one correct function?

   The functions all have the same zeros but different stretch factors.

   c. What would you tell Micheal, Tim, and Judy to correct their functions.

   Michael & Tim need a negative and Judy needs to change the signs

   d. How many possible functions can you write to represent the given characteristics? Explain your reasoning.

   An infinite number are possible

2. For a quadratic function written in factored form $f(x) = a(x - r_1)(x - r_2)$:

   a. what does the sign of the variable a tell you about the graph?

   If a is positive, the graph faces up
   If a is negative, the graph faces down

   b. what do the variables r_1 and r_2 tell you about the graph?

   r_1 and r_2 are the zeros, or x-intercepts

3. Use the given information to write a quadratic function in factored form, $f(x) = a(x - r_1)(x - r_2)$.

   a. The parabola opens upward and the zeros are (2, 0) and (4, 0).

   $f(x) = +(x-2)(x-4)$

   b. The parabola opens downward and the zeros are (−3, 0) and (1, 0).

   $f(x) = -(x+3)(x-1)$

   c. The parabola opens downward and the zeros are (0, 0) and (5, 0).

   $f(x) = -(x-0)(x-5)$ or $f(x) = -x(x-$

   d. The parabola opens upward and the zeros are (−2.5, 0) and (4.3, 0).

   $f(x) = +(x+2.5)(x-4.3)$

Station 5: I can solve a quadratic equation with a perfect square trinomial by factoring.

First multiply these factors together to find the pattern.

1) $(x+4)^2$ or $(x+4)(x+4)$ $x(x+4)+4(x+4) \rightarrow x^2+4x+4x+16 \rightarrow x^2+8x+16$
2) $(m-3)^2$ or $(m-3)(m-3)$ $m(m-3)-3(m-3) \rightarrow m^2-3m-3m+9 \rightarrow m^2-6m+9$
3) $(2p+5)^2$ $2p(2p+5)+5(2p+5) \rightarrow 4p^2+10p+10p+25 \rightarrow 4p^2+20p+25$
4) $(3x-1)^2$ $3x(3x-1)-1(3x-1) \rightarrow 9x^2-3x-3x+1 \rightarrow 9x^2-6x+1$

What patterns do you notice between the factors and the products?

There are alway two identical terms in the middle. The first term is squared and the last term is squared

5) Use the patterns to write $9z^2+30z+25$ in factored form.

$9z^2+15z+15z+25$ $(3z+5)(3z+5)$ or $(3z+5)^2$

6) Solve the following equation by factoring the perfect square trinomial on the left then square rooting: $9z^2+30z+25= 49$

$$(3z+5)^2 = 49$$

$$\sqrt{(3z+5)^2} = \pm\sqrt{49}$$

$3z+5=7$ or $3z+5=-7$

$3z=2$ $3z=-12$

$z=\frac{2}{3}$ or $z=-4$

Station 6: I can solve a quadratic equation using the quadratic formula.

For a quadratic equation of the form $ax^2 + bx + c = 0$, the solutions can be calculated using the **Quadratic Formula** $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Before using the Quadratic Formula, be sure to write the quadratic function in standard form: $ax^2 + bx + c$.

Consider the function $f(x) = -4x^2 - 40x - 99$.

First, determine the values of a, b, and c.

$a = -4$, $b = -40$, $c = -99$

Next, substitute the values into the Quadratic Formula.

$$x = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(-4)(-99)}}{2(-4)}$$

Then, simplify to determine the zeros of the function.

$$x = \frac{40 \pm \sqrt{1600 - 1584}}{-8}$$

$$x = \frac{40 \pm \sqrt{16}}{-8}$$

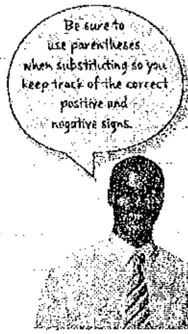
$$x = \frac{40 \pm 4}{-8}$$

$$x = \frac{40 + 4}{-8} \text{ or } x = \frac{40 - 4}{-8}$$

$$x = \frac{44}{-8} \text{ or } x = \frac{36}{-8}$$

$$x = -5.5 \text{ or } x = -4.5$$

The zeros of the function $f(x) = -4x^2 - 40x - 99$ are -5.5 and -4.5.



1. Javier was determining the exact zeros for $f(x) = x^2 - 14x + 19$. His work is shown.

Javier

$f(x) = x^2 - 14x + 19$

$a = 1, b = -14, c = 19$

$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(19)}}{2(1)}$$

$$x = \frac{14 \pm \sqrt{196 - 76}}{2}$$

$$x = \frac{14 \pm \sqrt{120}}{2}$$

$$x = \frac{14 \pm \sqrt{30 \cdot 4}}{2}$$

$$x = \frac{14 \pm 2\sqrt{30}}{2}$$

$$x = 7 \pm 2\sqrt{30}$$

a. Identify the error Javier made when determining the zeros.

He did not reduce both parts of the fraction.

b. Determine the correct zeros of the function.

$$X = \frac{14 \pm 2\sqrt{30}}{2}$$

$$X = \frac{14}{2} \pm \frac{2\sqrt{30}}{2}$$

$$X = 7 \pm \sqrt{30}$$

2. Use the Quadratic Formula to determine the zeros for each function given. This time, leave your solutions in exact form.

a. $f(x) = -2x^2 - 3x + 7$

$a = -2$
$b = -3$
$c = 7$

$$X = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$X = \frac{-(-3)}{2(-2)} \pm \frac{\sqrt{(-3)^2 - 4(-2)(7)}}{2(-2)}$$

$$X = \frac{3}{-4} \pm \frac{\sqrt{9 + 56}}{-4}$$

$$X = \frac{-3}{4} \pm \frac{\sqrt{65}}{-4}$$

b. $r(x) = -3x^2 + 19x - 7$

$a = -3$
$b = 19$
$c = -7$

$$X = \frac{-b}{2a} \pm \frac{\sqrt{b^2 - 4ac}}{2a}$$

$$= \frac{-(19)}{2(-3)} \pm \frac{\sqrt{(19)^2 - 4(-3)}}{2(-3)}$$

$$= \frac{-19}{-6} \pm \frac{\sqrt{361 - 84}}{-6}$$

$$= \frac{19}{6} \pm \frac{\sqrt{277}}{-6}$$

$$= \frac{19}{6} \pm \frac{\sqrt{277}}{6}$$

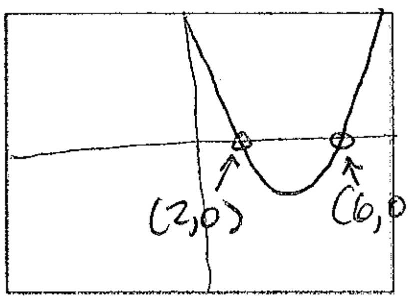
Station 7: I can solve a quadratic equation using a calculator.

The x-intercepts of a graph of a quadratic function are also called the **zeros** of the quadratic function.

> You can use a graphing calculator to determine the zeros of a quadratic function.
>
> **Step 1:** Press **2ND** and then **CALC**. Select **2: zero**.
>
> **Step 2:** Determine the left and right bounds for each point that appears to be a zero. Then press **ENTER**.

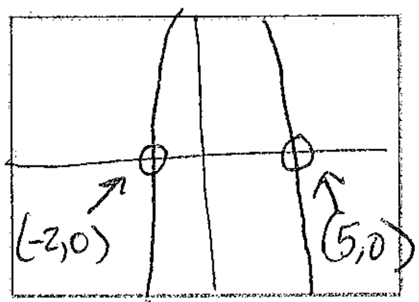
a. $h(x) = x^2 - 8x + 12$

zeros: (2,0) and (6,0)



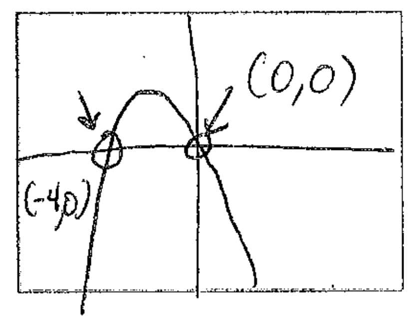
b. $r(x) = -2x^2 + 6x + 20$

zeros: (-2,0) and (5,0)



c. $w(x) = -x^2 - 4x$

zeros (-4,0) (0,0)



Station 8: I can complete the square to solve a quadratic equation.

A perfect square trinomial is the result of squaring a binomial. For example,

$$(2x+3)^2 = (2x+3)(2x+3)$$
$$= 2x(2x+3) + 3(2x+3)$$
$$= 4x^2 + 6x + 6x + 9$$
$$= 4x^2 + 12x + 9$$

The pattern is always the same with a perfect square trinomial:

The first term is a perfect square: $4x^2 = (2x)^2$

The last term is a perfect square: $9 = 3^2$

And the middle term is twice the product of the roots of the first and last: $12x = 2 \cdot (2x) \cdot 3$

So, $(ax+b)^2 = a^2x^2 + 2axb + b^2$

Use this pattern to determine the unknown value needed to make each expression a perfect square trinomial. Then write the trinomial as a binomial squared.

a. $x^2 - 8x +$ 16 = $(x-4)^2$

$\frac{-8x}{2x} = \frac{2ax \cdot b}{2x}$

$-4 = a \cdot b$

We know a = 1
so b = −4

$(-4)^2 = 16$

b. $x^2 + 5x +$ 6.25 = $(x+2.5)^2$

$\frac{5x}{2x} = \frac{2ax\,b}{2x}$

$2.5 = a \cdot b$

when know a = 1
so b = 2.5

$(2.5)^2 = 6.25$

c. $x^2 -$ 20x + 100 = $(x-10)^2$

$b^2 = 100$
$b = \pm\sqrt{100}$
$b = \pm 10$

We know the middle term must be negative
so b = −10

middle term =
$2ax\,b$
$2 \cdot 1 \cdot x \cdot (-10)$
$-20x$

d. $x^2 +$ 24x + 144 = $(x+12)^2$

$b^2 = 144$
$b = \pm\sqrt{144}$
$b = \pm 12$

We know the middle term must be positive
so b = 12

middle term =
$2ax\,b$
$2 \cdot 1 \cdot x \cdot 12$
$24x$

So how does completing the square help when trying to determine the roots of a quadratic equation that cannot be factored? Let's take a look.

Determine the roots of the equation $x^2 - 4x + 2 = 0$.

Isolate $x^2 - 4x$. You can make this into a perfect square trinomial.	$x^2 - 4x + 2 - 2 = 0 - 2$ $x^2 - 4x = -2$
Determine the constant term that would complete the square. Add this term to both sides of the equation.	$x^2 - 4x + \_\_\_\_ = -2 + \_\_\_\_$ $x^2 - 4x + 4 = -2 + 4$ $x^2 - 4x + 4 = 2$
Factor the left side of the equation.	$(x - 2)^2 = 2$
Determine the square root of each side of the equation.	$\sqrt{(x-2)^2} = \pm\sqrt{2}$ $x - 2 = \pm\sqrt{2}$
Set the factor of the perfect square trinomial equal to each of the square roots of the constant. Solve for x.	$x - 2 = \pm\sqrt{2}$ $x - 2 = \sqrt{2}$ or $x - 2 = -\sqrt{2}$ $x = 2 + \sqrt{2}$ or $x = 2 - \sqrt{2}$ $x \approx 3.414$ or $x \approx 0.5858$ The roots are approximately 3.41 and 0.59.

. Determine the roots of each equation by completing the square.

a. $x^2 - 6x + 4 = 0$

$a = 1$ $\quad \frac{2a \times b}{2x} = \frac{-6x}{2x}$

$b = ?$

$ab = -3$

$b = -3$

$(-3)^2 = 9$

$x^2 - 6x = -4$

$x^2 - 6x + \underline{9} = -4 + \underline{9}$

$(x-3)^2 = 5$

$\sqrt{(x-3)^2} = \pm\sqrt{5}$

$x - 3 = \sqrt{5}$ or $x - 3 = -\sqrt{5}$

$x = 3 + \sqrt{5}$ or $x = 3 - \sqrt{5}$

b. $x^2 - 12x + 6 = 0$

$a = 1$ $\quad \frac{2a \times b}{2x} = \frac{-12x}{2x}$

$b = ?$

$ab = -6$

$b = -6$

$(-6)^2 = 36$

$x^2 - 12x = -6$

$x^2 - 12x + \underline{36} = -6 + \underline{36}$

$(x-6)^2 = 30$

$\sqrt{(x-6)^2} = \pm\sqrt{30}$

$x - 6 = \pm\sqrt{30}$

$x - 6 = \sqrt{30}$ or $x - 6 = -\sqrt{30}$

$x = 6 + \sqrt{30}$ or $x = 6 - \sqrt{30}$

Station 9: I can complete the square to write a quadratic equation in vertex form.

You can identify the axis of symmetry and the vertex of any quadratic function written in standard form by completing the square.

$$y = ax^2 + bx + c$$

Step 1: $y - c = ax^2 + bx$

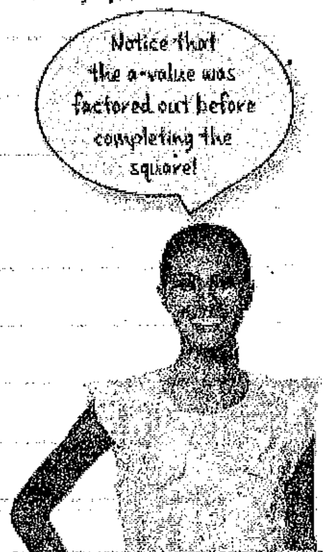
Step 2: $y - c = a\left(x^2 + \frac{b}{a}x\right)$

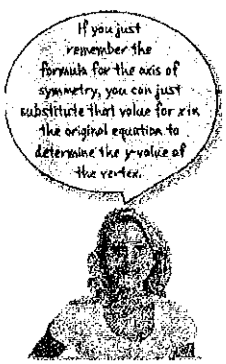
Step 3: $y - c + a\left(\frac{b}{2a}\right)^2 = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right)$

Step 4: $y - c + \frac{b^2}{4a} = a\left(x + \frac{b}{2a}\right)^2$

Step 5: $y - c + \frac{b^2}{4a} = a\left(x + \frac{b}{2a}\right)^2$

Step 6: $y = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$





1) Given a quadratic function in the form $y = ax^2 + bx + c$

a. identify the axis of symmetry.

$$x = -\frac{b}{2a}$$

b. identify the location of the vertex.

$$\left(-\frac{b}{2a}, c - \frac{b^2}{4a}\right)$$

2) Rewrite each quadratic equation in vertex form. Then identify the axis of symmetry and the location of the vertex in each.

a. $y = x^2 + 8x - 9$

$a = 1$
$b = 8$
$c = 9$

$\frac{8}{2} = 4$
$4^2 = 16$

$y + 9 = x^2 + 8x$

$y + 9 + \underline{16} = x^2 + 8x + \underline{16}$

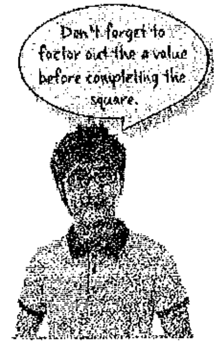
$y + 25 = (x + 4)^2$

$y = (x + 4)^2 - 25$

a.o.s. $x = -4$

vertex $(-4, -25)$

b. $y = 3x^2 + 2x - 1$



$y + 1 = 3x^2 + 2x$

$y + 1 = 3\left(x^2 + \frac{2}{3}x\right)$

$y + 1 + \underline{\frac{1}{3}} = 3\left(x^2 + \frac{2}{3}x + \underline{\frac{1}{9}}\right)$

$y + \frac{4}{3} = 3\left(x + \frac{1}{3}\right)^2$

$y = 3\left(x + \frac{1}{3}\right)^2 - \frac{4}{3}$

a.o.s. $x = -\frac{1}{3}$

vertex $\left(-\frac{1}{3}, -\frac{4}{3}\right)$

$\frac{\frac{2}{3}}{2} = \frac{1}{3}$

$\left(\frac{1}{3}\right)^2 = \frac{1}{9}$

$3\left(\frac{1}{9}\right) = \frac{1}{3}$

$1 = \frac{3}{3}$ so $1 + \frac{1}{3} = \frac{3}{3} + \frac{1}{3} = \frac{4}{3}$