

# Station 1: I can factor a trinomial where $a = 1$ .

Factor each quadratic expression.

a.  $x^2 + 8x + 15 =$  \_\_\_\_\_

$x^2 - 8x + 15 =$  \_\_\_\_\_

$x^2 + 2x - 15 =$  \_\_\_\_\_

$x^2 - 2x - 15 =$  \_\_\_\_\_

b.  $x^2 + 10x + 24 =$  \_\_\_\_\_

$x^2 - 10x + 24 =$  \_\_\_\_\_

$x^2 + 2x - 24 =$  \_\_\_\_\_

$x^2 - 2x - 24 =$  \_\_\_\_\_

**Station 1:** I can factor a trinomial where  $a \neq 1$ .

a) Factor  $3y^2 - 8y - 3$

b) Factor  $4m^2 + 13m + 3$

## Station 2: I can factor the difference of two squares.

First multiply these factors:

1)  $(x-4)(x+4)$

2)  $(x+3)(x-3)$

3)  $(3x-1)(3x+1)$

4)  $(2x^2+3)(2x^2-3)$

5) What pattern do you notice is always true when you multiply  $(a+b)(a-b)$ ?

Now use the pattern in reverse:

6) Factor  $9x^2-25$

7) Factor  $16-z^4$

# Station 3: I can solve a quadratic equation by factoring.

The solutions to a quadratic equation are called *roots*. The **roots** indicate where the graph of a quadratic equation crosses the  $x$ -axis. So, roots, zeros, and  $x$ -intercepts are all related.

To calculate the roots of a quadratic equation using factoring:

- Perform transformations so that one side of the equation is equal to zero.
- Factor the quadratic expression on the other side of the equation.
- Set each factor equal to zero.
- Solve the resulting equations for the roots. Check each solution in the original equation.

You can calculate the roots for the quadratic equation  $x^2 - 4x = -3$ .

$$x^2 - 4x = -3$$
$$x^2 - 4x + 3 = -3 + 3$$
$$x^2 - 4x + 3 = 0$$
$$(x - 3)(x - 1) = 0$$
$$(x - 3) = 0 \quad \text{or} \quad (x - 1) = 0$$
$$x - 3 + 3 = 0 + 3 \quad \text{or} \quad x - 1 + 1 = 0 + 1$$
$$x = 3 \quad \text{or} \quad x = 1$$

Check:

$$x^2 - 4x = (3)^2 - 4(3) = 9 - 12 = -3$$
$$x^2 - 4x = (1)^2 - 4(1) = 1 - 4 = -3$$

Solve by factoring:

a)  $3x^2 - 22x + 7 = 0$

b)  $w^2 + 8w = -7$

## Station 5: I can solve a quadratic equation with a perfect square trinomial by factoring.

First multiply these factors together to find the pattern.

1)  $(x+4)^2$  or  $(x+4)(x+4)$

2)  $(m-3)^2$  or  $(m-3)(m-3)$

3)  $(2p+5)^2$

4)  $(3x-1)^2$

5) What patterns do you notice between the factors and the products?

6) Use the patterns to write  $9z^2+30z+25$  in factored form.

7) Solve the following equation by factoring the perfect square trinomial on the left then square rooting:  $9z^2+30z+25=0$

# Station 4: I can identify the axis of symmetry without a calculator.

Standard Form of a Quadratic Function:  $y = ax^2 + bx + c$

Remember, when  
a quadratic equation is  
written in standard form,  
the axis of symmetry is  
 $x = -\frac{b}{2a}$ .



Given each quadratic equation, write the equation for the axis of symmetry of the graph.

a)  $f(x) = 6x^2 + 24x - 10$

b)  $h(t) = -16t^2 + 120t + 3$

## Station 4: I can sketch a graph of a quadratic without my calculator.

1. A group of students are working together on the problem shown.

Write a quadratic function in factored form to represent a parabola that opens downward and has zeros at (4, 0) and (-1, 0).

 **Maureen**

My function is

$$k(x) = -(x - 4)(x + 1).$$

 **Micheal**


My function is

$$d(x) = \frac{1}{2}(x - 4)(x + 1).$$

 **Tim**

My function is

$$m(x) = 2(x - 4)(x + 1).$$

 **Tom**

My function is

$$k(x) = -2(x - 4)(x + 1).$$

 **Dianne**

My function is

$$f(x) = -0.5(x - 4)(x + 1).$$

 **Judy**

My function is

$$f(x) = -(x + 4)(x - 1).$$

2. For a quadratic function written in factored form  $f(x) = a(x - r_1)(x - r_2)$ :

a. what does the sign of the variable  $a$  tell you about the graph?

b. what do the variables  $r_1$  and  $r_2$  tell you about the graph?

3. Use the given information to write a quadratic function in factored form,  $f(x) = a(x - r_1)(x - r_2)$ .

a. The parabola opens upward and the zeros are (2, 0) and (4, 0).

b. The parabola opens downward and the zeros are (-3, 0) and (1, 0).

c. The parabola opens downward and the zeros are (0, 0) and (5, 0).

d. The parabola opens upward and the zeros are (-2.5, 0) and (4.3, 0).

a. Use your graphing calculator to graph each student's function. What are the similarities among all the graphs? What are the differences among the graphs.

b. How is it possible to have more than one correct function?

c. What would you tell Micheal, Tim, and Judy to correct their functions.

d. How many possible functions can you write to represent the given characteristics? Explain your reasoning.

# Station 7: I can solve a quadratic equation using a calculator.

The  $x$ -intercepts of a graph of a quadratic function are also called the **zeros** of the quadratic function. First, set the function equal to zero. Then...



You can use a graphing calculator to determine the zeros of a quadratic function.

**Step 1:** Press **2ND** and then **CALC**. Select **2: zero**.

**Step 2:** Determine the left and right bounds for each point that appears to be a zero. Then press **ENTER**.

a.  $h(x) = x^2 - 8x + 12$

zeros: \_\_\_\_\_



b.  $r(x) = -2x^2 + 6x + 20$

zeros: \_\_\_\_\_



c.  $w(x) = -x^2 - 4x$





# Station 6: I can solve a quadratic equation using the quadratic formula.

For a quadratic equation of the form  $ax^2 + bx + c = 0$ , the solutions can be calculated using the

**Quadratic Formula**  $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ .

Before using the Quadratic Formula, be sure to write the quadratic function in standard form:  $ax^2 + bx + c$ .

Consider the function  $f(x) = -4x^2 - 40x - 99$ .

First, determine the values of  $a$ ,  $b$ , and  $c$ .  
 $a = -4$ ,  $b = -40$ ,  $c = -99$

Next, substitute the values into the Quadratic Formula.

$$x = \frac{-(-40) \pm \sqrt{(-40)^2 - 4(-4)(-99)}}{2(-4)}$$

Then, simplify to determine the zeros of the function.

$$x = \frac{40 \pm \sqrt{1600 - 1584}}{-8}$$

$$x = \frac{40 \pm \sqrt{16}}{-8}$$

$$x = \frac{40 \pm 4}{-8}$$


$$x = \frac{40+4}{-8} \text{ or } x = \frac{40-4}{-8}$$

$$x = \frac{44}{-8} \text{ or } x = \frac{36}{-8}$$

$$x = -5.5 \text{ or } x = -4.5$$

The zeros of the function  $f(x) = -4x^2 - 40x - 99$  are  $-5.5$  and  $-4.5$ .

Be sure to use parentheses when substituting so you keep track of the correct positive and negative signs.



1. Javier was determining the exact zeros for  $f(x) = x^2 - 14x + 19$ . His work is shown.

**Javier**

$$f(x) = x^2 - 14x + 19$$

$$a = 1, b = -14, c = 19$$

$$x = \frac{-(-14) \pm \sqrt{(-14)^2 - 4(1)(19)}}{2(1)}$$

$$x = \frac{14 \pm \sqrt{196 - 76}}{2}$$

$$x = \frac{14 \pm \sqrt{120}}{2}$$

$$x = \frac{14 \pm \sqrt{30} \cdot 4}{2}$$

$$x = \frac{14 \pm 2\sqrt{30}}{2}$$

$$x = 7 \pm 2\sqrt{30}$$

- a. Identify the error Javier made when determining the zeros.

- b. Determine the correct zeros of the function.

2. Use the Quadratic Formula to determine the zeros for each function given. This time, leave your solutions in exact form.

a.  $f(x) = -2x^2 - 3x + 7$

b.  $r(x) = -3x^2 + 19x - 7$

## Station 8: I can complete the square to solve a quadratic equation.

A perfect square trinomial is the result of squaring a binomial. For example,

$$\begin{aligned}(2x + 3)^2 &= (2x + 3)(2x + 3) \\ &= 2x(2x + 3) + 3(2x + 3) \\ &= 4x^2 + 6x + 6x + 9 \\ &= 4x^2 + 12x + 9\end{aligned}$$

The pattern is always the same with a perfect square trinomial:

The first term is a perfect square:  $4x^2 = (2x)^2$

The last term is a perfect square:  $9 = 3^2$

And the middle term is twice the product of the roots of the first and last:  $12x = 2 \cdot (2x) \cdot 3$

So,  $(ax + b)^2 = a^2x^2 + 2axb + b^2$

Use this pattern to determine the unknown value needed to make each expression a perfect square trinomial. Then write the trinomial as a binomial squared.

Example:  $x^2 + 6x + 9 = (x + 3)^2$

a.  $x^2 - 8x + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

b.  $x^2 + 5x + \underline{\hspace{2cm}} = \underline{\hspace{2cm}}$

c.  $x^2 - \underline{\hspace{2cm}} + 100 = \underline{\hspace{2cm}}$

d.  $x^2 + \underline{\hspace{2cm}} + 144 = \underline{\hspace{2cm}}$

So how does completing the square help when trying to determine the roots of a quadratic equation that cannot be factored? Let's take a look.

Determine the roots of the equation  $x^2 - 4x + 2 = 0$ .

Isolate  $x^2 - 4x$ . You can make this into a perfect square trinomial.

$$x^2 - 4x + 2 - 2 = 0 - 2$$

$$x^2 - 4x = -2$$

Determine the constant term that would complete the square.

$$x^2 - 4x + \underline{\quad} = -2 + \underline{\quad}$$

$$x^2 - 4x + 4 = -2 + 4$$

Add this term to both sides of the equation.

$$x^2 - 4x + 4 = 2$$

Factor the left side of the equation.

$$(x - 2)^2 = 2$$

Determine the square root of each side of the equation.

$$\sqrt{(x - 2)^2} = \pm\sqrt{2}$$

$$x - 2 = \pm\sqrt{2}$$

Set the factor of the perfect square trinomial equal to each

$$x - 2 = \pm\sqrt{2}$$

$$x - 2 = \sqrt{2} \quad \text{or} \quad x - 2 = -\sqrt{2}$$

of the square roots of the constant.

$$x = 2 + \sqrt{2} \quad \text{or} \quad x = 2 - \sqrt{2}$$

Solve for  $x$ .

$$x \approx 3.414 \quad \text{or} \quad x \approx 0.5858$$

The roots are approximately

3.41 and 0.59.

Determine the roots of each equation by completing the square.

a.  $x^2 - 6x + 4 = 0$

b.  $x^2 - 12x + 6 = 0$

# Station 9: I can complete the square to write a quadratic equation in vertex form.

You can identify the axis of symmetry and the vertex of any quadratic function written in standard form by completing the square.

$$y = ax^2 + bx + c$$

Step 1:  $y - c = ax^2 + bx$

Step 2:  $y - c = a\left(x^2 + \frac{b}{a}x\right)$


Step 3:  $y - c + a\left(\frac{b}{2a}\right)^2 = a\left(x^2 + \frac{b}{a}x + \left(\frac{b}{2a}\right)^2\right)$

Step 4:  $y - c + \frac{b^2}{4a} = a\left(x + \frac{b}{2a}\right)^2$

Step 5:  $y - c + \frac{b^2}{4a} = a\left(x + \frac{b}{2a}\right)^2$

Step 6:  $y = a\left(x + \frac{b}{2a}\right)^2 + \left(c - \frac{b^2}{4a}\right)$

Notice that the a-value was factored out before completing the square!



If you just remember the formula for the axis of symmetry, you can just substitute that value for  $x$  in the original equation to determine the  $y$ -value of the vertex.



- 1) Given a quadratic function in the form  $y = ax^2 + bx + c$
- identify the axis of symmetry.

- identify the location of the vertex.

Rewrite each quadratic equation in vertex form.

- 2) Then identify the axis of symmetry and the location of the vertex in each.

- $y = x^2 + 8x - 9$

- $y = 3x^2 + 2x - 1$