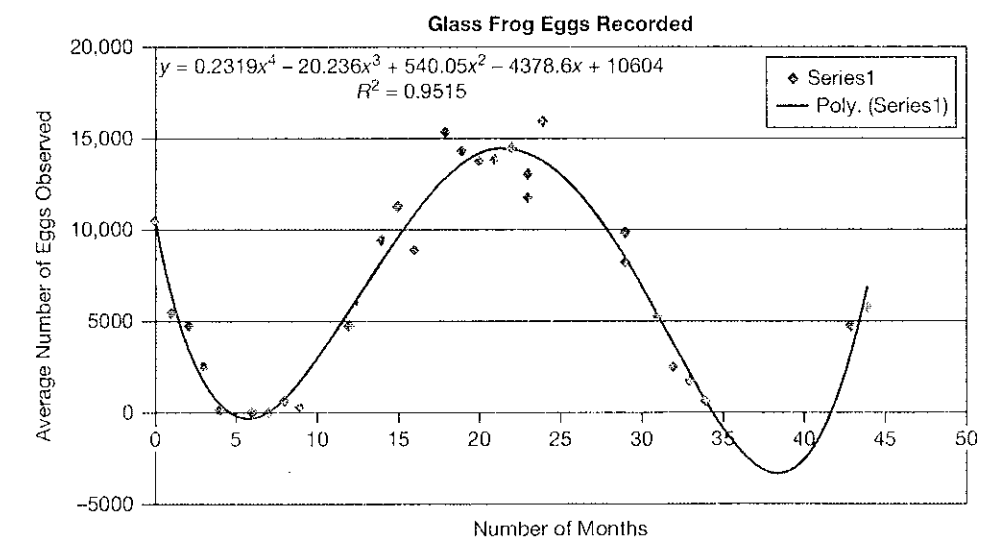**Math World vs. Real World**

The data shown represents the population of a rare, endangered species of frog called the glass frog. In order to better understand the glass frog's fertilization habits, scientists performed a study and recorded the average number of frog eggs over the span of 44 months.

Month of Study	Average Number of Glass Frog Eggs	Month of Study	Average Number of Glass Frog Eggs
0	10,534	19	14,330.5
1	5500	20	13,845.1
2	5033	21	13,893.1
3	2600	22	14,546.3
4	239.4	23	11,815.8
6	137.3	23	13,086.2
7	108.4	24	15,966.9
8	667.1	29	9904.4
9	387.4	29	8257.3
12	4813.1	31	5297.5
14	9539.5	32	2494.1
15	11,318.6	33	1805.4
16	8953.3	34	665
18	15,402.5	43	4813

The data has been plotted for you and a quartic regression was used to generate the polynomial function to best represent the data. The quartic regression option calculates the best-fit equation of the form $y = ax^4 + bx^3 + cx^2 + dx + e$.

Glass Frog Eggs Recorded

$y = 0.2319x^4 - 20.236x^3 + 540.05x^2 - 4378.6x + 10604$
$R^2 = 0.9515$

♦ Series1
— Poly. (Series1)

Average Number of Eggs Observed

Number of Months

1. Consider the graph and equation to answer each question.

   a. What is the domain and range of the study?

   b. Explain what the domain and range represent in the context of this problem.

   c. What is the domain and range of the function?

   d. At what month in the study were the most frog eggs observed? How many eggs were recorded?

   e. At what month in the study were the least frog eggs observed? How many eggs were recorded?

   f. If the study lasted for 50 months, how many frog eggs would there be according to the function?

   g. If the study lasted forever, how many eggs would there be according to the function?

   h. How many frog eggs appeared between months 35 and 40?

   i. At what month(s) of the study were there approximately 4800 glass frog eggs observed?

2. Use a graphing calculator to determine the x-intercepts of the function. What do the x-intercepts mean in the context of this problem situation?

3. State the end behavior of the function. Does this make sense in the context of this problem scenario? Explain your reasoning.

4. How many frog eggs.were observed at the beginning of the study? Explain the mathematical meaning of your answer.

5. Describe the interval when the frog's egg population is:

a. increasing.

b. decreasing.

PROBLEM A Polynomial is Born

So far in this chapter, you have learned a great deal about polynomial functions. You have learned about minimums, maximums, zeros, end behavior, and the general shapes of their graphs. Now, you will combine all that information to generalize the key characteristics for any degree polynomial.

Recall the definition of a relative maximum is the highest point in a particular section of a function's graph, and a relative minimum is the lowest point in a particular section of the graph. Similarly, the **absolute maximum** is the highest point in the entire graph, and the **absolute minimum** is the lowest point in the entire graph. The set of absolute maximums, absolute minimums, relative maximums, and relative minimums may also be referred to as **extrema**. The extrema are also called extreme points and extremum.

1. Consider the graph that represents the average number of glass frog eggs in Problem 1.

a. State all relative maximums and minimums.

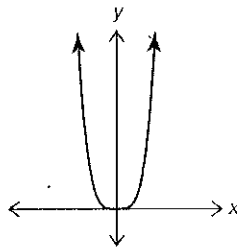**b.** State all absolute maximums and minimums.

c. Do the absolute minimums and/or maximums make sense in the context of this problem situation? Explain your reasoning.

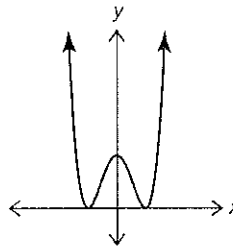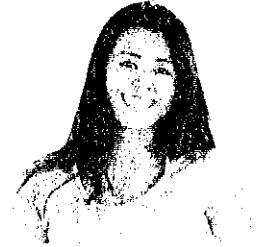**2.** Determine the number of extrema in each polynomial.

Don't forget to look for relationships!

$g_1(x) = x^4$ $g_2(x) = x^4 - 3x^2$

4ᵗʰ Degree Polynomials

Number of Extrema _____ _____

$f_1(x) = x^5$ $f_2(x) = x^5 + 4x^2$ $f_3(x) = x^5 - 5x^3 + 5x + 1.18$

5ᵗʰ Degree Polynomials
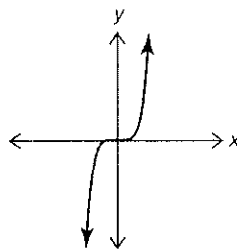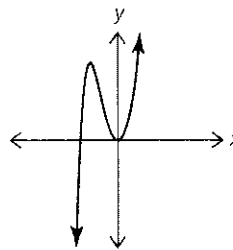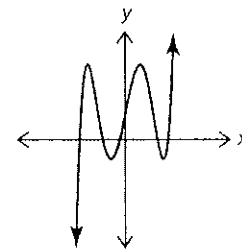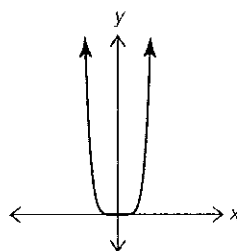
Number of Extrema _____ _____ _____

$h_1(x) = x^6$ $h_2(x) = x^6 - 3x^2$ $h_3(x) = 2x^6 - 13x^5 + 26x^4 - 7x^3 - 2$
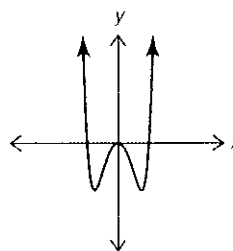
6ᵗʰ Degree Polynomials
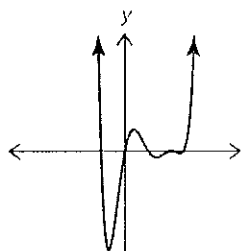
Number of Extrema _____ _____ _____

3. List any observations you notice about the possible number of extrema and the degree of the polynomial.

4. List the possible number of extrema for the each polynomial.

    a. 9th degree polynomial

    b. 18th degree polynomial

    c. nth degree odd polynomial

    d. nth degree even polynomial

Use the knowledge you gained about 4th, 5th, and 6th degree polynomials to answer these questions.

5. Choose the appropriate word from the box to complete each statement. Justify your answer with a sketch or explanation.

always	sometimes	never

    a. An odd degree function will _____ have absolute extrema.

    b. An even degree function will _____ have relative extrema.

    c. An even degree function will _____ have 3 or more relative extrema.

d. An even degree function will _____ have absolute extrema.

e. An odd degree function will _____ have relative extrema.

f. An odd degree function will _____ one have relative extrema.