PCFU-CP: Zeros C8-C10 Name: ___________________________________

1. Without a calculator identify the zeros of the following polynomial

$$y = x^3 - 2x^2 - 15x$$

To do this I will need to factor the polynomial. I used my factoring decision tree to determine the best method of factoring this polynomial.

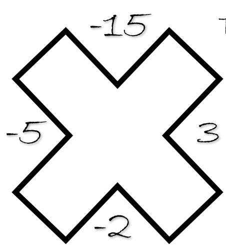
1st Question: Does it have a GCF? Yes! x can be factored out of each term in the polynomial so the GCF is x.

$$y = x(x^2 - 2x - 15)$$

2nd Question: How many terms does the resulting quadratic have? Three!

3rd Questions: Are the first and last terms perfect squares? No! **Does a=1?** Yes! Since a=1, I can use the diamond shortcut.

I know that a*c goes into the top of the diamond and b goes into the bottom of the diamond. Then I must find two integers that multiply to the top number and add to the bottom number.



The two integers that multiple to -15 and add to -2 are -5 and 3. Since a=1, I can put these integers directly into factored form.

$$y = x(x - 5)(x + 3)$$

Now that the polynomial is completely factored, I set each factor equal to zero and solve for x to find the zeros.

$$x = 0 \quad or \quad x - 5 = 0 \quad or \quad x + 3 = 0$$
$$x = 0 \qquad\quad x = 5 \qquad\quad x = -3$$

Additional Practice:

1) Without a calculator identify the zeros of the following polynomial: $y = 3x^4 + 3x^3 - 36x^2$

1st Question: Does it have a GCF?

a. What <u>number</u> goes into each term of the polynomial?
b. What shared <u>variable</u> does each term have?
c. What is the highest <u>exponent</u> that each term has?
d. Now, what is the <u>greatest common factor</u> of the terms of the polynomial?
e. Factor out the GCF.

2nd Question: How many terms does it have?

f. The remaining polynomial should be quadratic with three terms.

3rd Questions: Are the first and last terms perfect squares? Is a = 1? Is a ≠ 1?

g. Based on your answers above, factor the remaining quadratic.

h. Put all the factors together to make a "factored form" equation. Now set the equation equal to zero in place of y.

i. If the factors must multiply to equal zero, then one of the factors must be zero. Set each factor equal to zero then solve each individual equation.

j. You should have found three zeros. One of them has multiplicity. Which one? What is the multiplicity of that zero?

k. How will these zeros show up on the graph?

2) Build a polynomial in standard form with the following zeros. $x = 3$ *with a multiplicity of* 2 *and* $x = -1$.

I know that each zero gets its own factor. If r is a zero then (x-r) is a factor. Multiplicity of 2 means the root repeats. I will show this by squaring the factor that has that zero.

$$y = (x-3)^2(x--1)$$
$$y = (x-3)^2(x+1)$$

Now to get the polynomial in standard form I will need to multiply out the factors. I can use the distributive property to multiply two of them and then take the result and multiply with the third one, OR I can use the perfect square trinomial shortcut then the distributive property with the third factor. Finally, I combine like terms.

$$y = (x-3)(x-3)(x+1)$$
$$y = (x^2-6x+9)(x+1)$$
$$y = x(x^2-6x+9)+1(x^2-6x+9)$$
$$y = x^3-6x^2+9x+x^2-6x+9$$
$$y = x^3-5x^2+3x+9$$

Additional Practice:

2) Given a polynomial has zeros at x=2 and x=-1 with a multiplicity of 2, find:

a. A polynomial equation in factored form that will have these same zeros.

b. The standard form of the equation you just wrote. (This means you have to multiply it out. Be careful with the squared factor!)

1) Without a calculator identify the zeros of the following polynomial

$$y = 2x^3 + x^2 - 15x$$

To do this I will need to factor the polynomial. I used my factoring decision tree to determine the best method of factoring this polynomial.

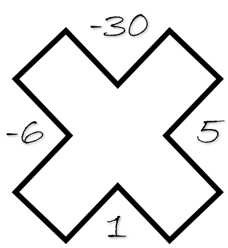
1st Question: Does it have a GCF? *Yes! x can be factored out of each term in the polynomial so the GCF is x.*

$$y = x(2x^2 + x - 15)$$

2nd Question: How many terms does the resulting quadratic have? *Three!*

3rd Questions: Are the first and last terms perfect squares? *No!* **Does a=1?** *No! Since a=2, I need to use the diamond and reverse the distributive property.*

*I know that a*c goes into the top of the diamond and b goes into the bottom of the diamond. Then I must find two integers that multiply to the top number and add to the bottom number.*



The two integers that multiply to -30 and add to 1 are -6 and 5. I use these coefficients to split the middle term and pair the terms.	$2x^2 + \mathrm{x} - 15$ $(2x^2 - 6\mathrm{x}) + (5\mathrm{x} - 15)$
Then I factor a GCF out of each pair and make sure the parenthesis match.	$2x(x - 3) + 5(x - 3)$
Last, I create a binomial factor from the GCF's and the other binomial factor is what is in the parenthesis.	$(2x + 5)(x - 3)$

$$y = x(2x + 5)(x - 3)$$

Now that the polynomial is completely factored, I set each factor equal to zero and solve for x to find the zeros.

$$x = 0 \quad or \quad 2x - 5 = 0 \quad or \quad x + 3 = 0$$

$$x = 0 \qquad x = \frac{5}{2} \qquad x = -3$$

Additional Practice:

1) Without a calculator identify the zeros of the following polynomial: $y = 3x^4 + 3x^3 - 36x^2$

1st Question: Does it have a GCF?

a. What <u>number</u> goes into each term of the polynomial?
b. What shared <u>variable</u> does each term have?
c. What is the highest <u>exponent</u> that each term has?
d. Now, what is the <u>greatest common factor</u> of the terms of the polynomial?
e. Factor out the GCF.

2nd Question: How many terms does it have?

f. The remaining polynomial should be quadratic with three terms.

3rd Questions: Are the first and last terms perfect squares? Is a = 1? Is a ≠ 1?

g. Based on your answers above, factor the remaining quadratic.

h. Put all the factors together to make a "factored form" equation. Now set the equation equal to zero in place of y.

i. If the factors must multiply to equal zero, then one of the factors must be zero. Set each factor equal to zero then solve each individual equation.

j. You should have found three zeros. One of them has multiplicity. Which one? What is the multiplicity of that zero?

k. How will these zeros show up on the graph?

2) Build a polynomial in standard form with the following zeros. $x = 3 \ with\ a\ multiplicity\ of\ 2\ and\ x = \frac{1}{2}$.

I know that each zero gets its own factor. If r is a zero then (x-r) is a factor. Multiplicity of 2 means the root repeats. I will show this by squaring the factor that has that zero. Since the other zero is a fraction, I can use the numerator as my r-value and the denominator as the coefficient on x.

$$y = (x-3)^2(2x-1)$$

Now to get the polynomial in standard form I will need to multiply out the factors. I can use the distributive property to multiply two of them and then take the result and multiply with the third one, OR I can use the perfect square trinomial shortcut then the distributive property with the third factor. Finally, I combine like terms.

$$y = (x-3)(x-3)(2x-1)$$
$$y = (x^2 - 6x + 9)(2x-1)$$
$$y = 2x(x^2 - 6x + 9) - 1(x^2 - 6x + 9)$$
$$y = 2x^3 - 12x^2 + 18x - x^2 + 6x - 9$$
$$y = 2x^3 - 13x^2 + 24x - 9$$

Additional Practice:

2) Given a polynomial has zeros at x=2 and x=-1 with a multiplicity of 2, find:

a. A polynomial equation in factored form that will have these same zeros.

b. The standard form of the equation you just wrote. (This means you have to multiply it out. Be careful with the squared factor!)