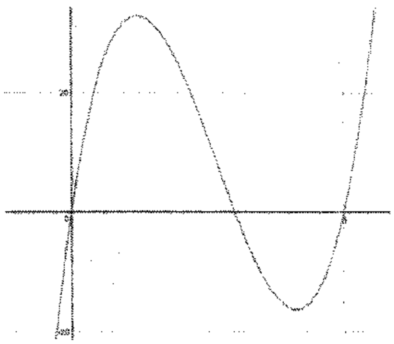
Polynomials Unit C Test Review - CP
NCM3

Name Key 2018 - 2019 #

I can represent cubic functions using words, tables, graphs & equations. I can determine domain and range of cubic functions in context. I can connect characteristics to factors. (C2, C3)

1. The volume $V(x)$ of a box is defined by the function: $V(x) = x(6-2x)(10-2x)$, where each factor represents a dimension of the box. What is the domain and range of the function? What is the domain and range in context?



a) Function (theoretical):

Domain: $\mathbb{R}$ $(-\infty, \infty)$

Range: $\mathbb{R}$ $(-\infty, \infty)$

b) Context (practical):

Domain: $(0, 3)$

Range: $(0, 32.835)$ ← approx.

c) What do the practical domain and range mean for this problem?

Practical Domain represent lengths possible to cut from the corner of the paper to create heights for the box. Practical Range represents possible volumes the box could have.

I can determine if a graph or an equation represents an even or odd function. (C4)

2. Evaluate for the criteria that apply determine whether each is an even or odd function.

a) $f(x) = 5x^4 - 7x^2 + 9$

i) Describe the exponents: on variables — all even (4, 2, 0)

ii) $f(-x) = 5(-x)^4 - 7(-x)^2 + 9 = 5x^4 - 7x^2 + 9 = f(x)$

iii) Is the function odd, even, or neither? $\boxed{\text{even}}$

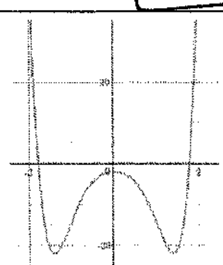
b) $c(x) = 6x^5 - 4x^2 + 2$

i) Describe the exponents: on variables — mixture of odd/even (5, 2, 0)

ii) $f(-x) = 6(-x)^5 - 4(-x)^2 + 2 = -6x^5 + 4x^2 + 2 \neq f(x) \neq -f(x)$

iii) Is the function odd, even, or neither? $\boxed{\text{neither}}$

c)

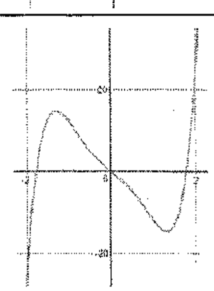


i) What type (if any) of symmetry does it have? y-axis

ii) $f(-x) = f(x)$

iii) Is the function odd, even, or neither? $\boxed{\text{even}}$

d)



i) What type (if any) of symmetry does it have? origin

ii) $f(-x) = -f(x)$

iii) Is the function odd, even, or neither? $\boxed{\text{odd}}$

I can determine the possible number of relative extrema, number absolute extrema, the number of possible zeros by looking at the degree of a polynomial. (C5)

3. State the number(s) of possible relative extrema, number of absolute extrema & possible number(s) of zeros for each function:

(a) $f(x)$ is a 5[th] degree polynomial: Relative 4,2, or 0 Absolute 0 Zeros 5

(b) $f(x)$ is a 6[th] degree polynomial: Relative 5, 3, 1 Absolute 1 Zeros 6

I can find extrema, intervals of increase or decrease, end behaviors, and possible number of zeros based on the degree of the polynomial. (C6)

4. $f(x) = x^4 - 52x^2 + 576$ Use window: [-10, 10, 1, -100, 700, 1]

Number of Absolute Extrema: 1

Number of Relative Extrema: 3

Coordinates of minimum(s): ≈ (-5.1, -100) & (5.1, -100)

Coordinates of maximum(s): (0, 576)

* watch out for fake-out scientific notation on calc!!

Intervals of Increase: $(-5.1, 0) \cup (5.1, \infty)$

Intervals of Decrease: $(-\infty, -5.1) \cup (0, 5.1)$

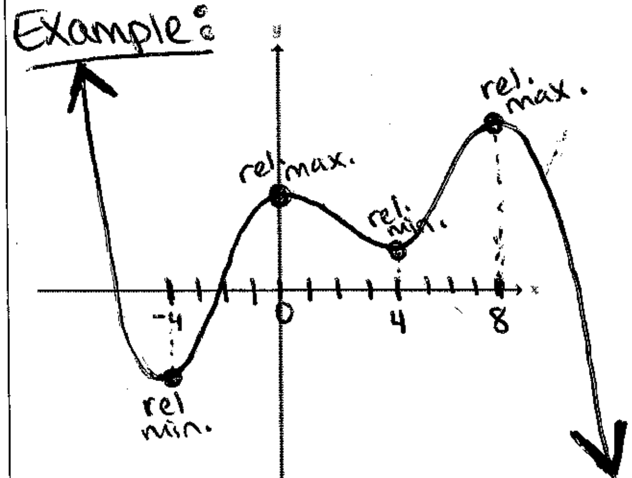
End Behavior: As $x \to -\infty$, $y \to \infty$, As $x \to \infty$, $y \to \infty$

I can draw a graph of a polynomial function given the characteristics. (C7)

5. As $x \to -\infty$, $y \to \infty$, As $x \to \infty$, $y \to -\infty$. Relative minimums at $x = -4,\ 4$; relative maximums at $x = 0,\ 8$

left → up right → down

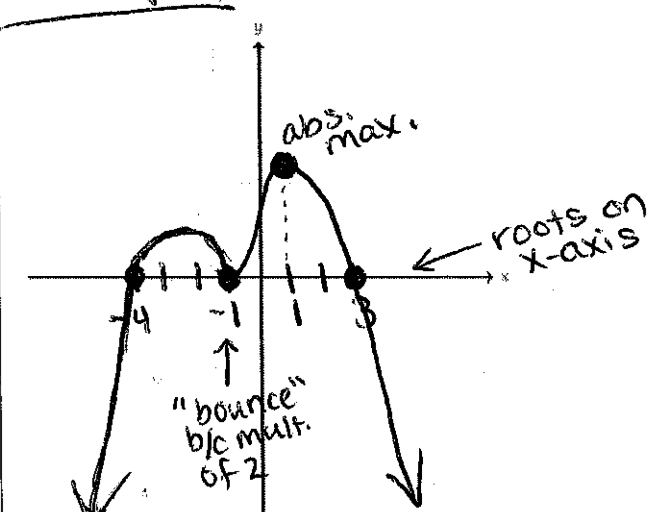
Example:



Answers will vary for y-values of max/min points, but x-values & behavior/shape should be the same

6. Roots: $x = -4$, 3, and -1 with multiplicity 2. Absolute maximum at $x = 1$

Example:



Answers will vary for y-value of abs. max point, but x-values & behavior/shape should be the same.

I can understand the fundamental theorem of algebra. (C8)

Circle all that are possible.

7. *Quartic Function* (degree 4)

(a) 1 real root, 4 imaginary roots

(b) 3 real roots, 1 imaginary root

(c) 4 real roots, 0 imaginary roots

(d) 2 real roots, 2 imaginary roots

(e) 5 real roots, 0 imaginary roots

8. *Quintic Function* (degree 5)

(a) 7 real roots, 0 imaginary roots

(b) 3 real roots, 2 imaginary roots

(c) 4 real roots, 1 imaginary root

(d) 1 real root, 4 imaginary roots

(e) 2 real roots, 3 imaginary roots

I can write polynomials in standard and factored form and find their zeros. (C9, C10)

9. Write in standard form:

$$(x+4)^2(x-3)$$

$$(x+4)(x+4)(x-3)$$
$$x^2+4x+4x+16$$
$$(x^2+8x+16)(x-3)$$
$$x^3-3x^2+8x^2-24x+16x-48$$
$$\boxed{x^3+5x^2-8x-48}$$

10. Write in factored form: $x^5-4x^4+4x^3$

GCF!

$$x^3(x^2-4x+4)$$

4, −2, −2, −4

$$\boxed{x^3(x-2)(x-2)}$$

OR $\boxed{x^3(x-2)^2}$

11. Write in factored form: $36x^3-3x^2-18x$

GCF!

$$3x(12x^2-x-6)$$

−72, −9, 8, −1

$$3x[12x^2-9x+8x-6]$$
$$3x[3x(4x-3)+2(4x-3)]$$
$$\boxed{3x(3x+2)(4x-3)}$$

12. Find the zeros algebraically and indicate multiplicity:

$$c(x)=-2(x+3)^3(x-2)^2(x+2)$$

$x+3=0$, $-3\ \ -3$, $x=-3$

$x-2=0$, $+2\ \ +2$, $x=2$

$x+2=0$, $-2\ \ -2$, $x=-2$

Zeros: $x=-3$ with mult. of three
$x=2$ with mult. of two
$x=-2$

I can determine factors of a polynomial and divide a polynomial by a binomial using long division. (C11, C12)

13. Use long division to determine if $5x+7$ is a factor of $5x^3-28x^2+x+70$.

$$\begin{array}{r} x^2-7x+10 \\ 5x+7\,\big)\,\overline{5x^3-28x^2+x+70} \\ -(5x^3+7x^2) \\ \hline -35x^2+x \\ +35x^2+49x \\ \hline 50x+70 \\ -(50x+70) \end{array}$$

R 0

Is $5x+7$ a factor of $5x^3-28x+x+70$?

Yes, b/c R = 0.

14. Use long division to evaluate

$$4x^4-19x^3+24x^2-49x-10 \div 4x-3.$$

$$\begin{array}{r} x^3-4x^2+3x-10 \\ 4x-3\,\big)\,\overline{4x^4-19x^3+24x^2-49x-10} \\ -(4x^4-3x^3) \\ \hline -16x^3+24x^2 \\ +16x^3-12x^2 \\ \hline 12x^2-49x \\ -(12x^2-9x) \\ \hline -40x-10 \\ +40x-30 \\ \hline -40 \end{array}$$

$$\boxed{x^3-4x^2+3x-10,\ R\ -40}$$

I can divide a polynomial by a binomial using synthetic division. I can write the dividend as the product of the divisor and the quotient plus the remainder. (C13)

15. Use synthetic division to determine if $x - 3$ is a factor of $3x^4 - 2x^2 - 70x - 20$.

$$\begin{array}{r|rrrrr} 3 & 3 & 0 & -2 & -70 & -20 \\ & \downarrow & 9 & 27 & 75 & 15 \\ \hline & 3 & 9 & 25 & 5 & \boxed{-5} \end{array}$$

Is $x - 3$ a factor of $3x^4 - 2x^2 - 70x - 20$?

No, b/c $R \neq 0$.

16. Use synthetic division to evaluate $2x^4 + 16x^3 + 34x^2 - 4x + 48 \div x + 4$.

$$\begin{array}{r|rrrrr} -4 & 2 & 16 & 34 & -4 & 48 \\ & \downarrow & -8 & -32 & -8 & 48 \\ \hline & 2 & 8 & 2 & -12 & \boxed{96} \end{array}$$

$\boxed{2x^3+8x^2+2x-12,\ R\ 96}$

17. Use your answer in #15 to write the polynomial as a product of the divisor and the quotient plus the remainder.

$$3x^4-2x^2-70x-20 = \underbrace{(x-3)}_{\text{divisor}}\underbrace{(3x^3+9x^2+25x+5)}_{\text{quotient}} + \underbrace{-5}_{\text{remainder}}$$

I can use the remainder theorem to evaluate a polynomial for a given value. I can use the remainder theorem to find a missing value. (C14)

18. Given $\frac{P(x)}{x-4} = x^2 - 5x + 6\ R\ 42$ what is $P(4)$? Show work or explain your answer.

Since +4 is the root of the divisor $x-4$, the Remainder Thm. says that $P(4)$ = Remainder, so $\boxed{P(4)=42}$

19. If $\frac{x^4+2x^2+ax+4}{x-2} = x^3 + 2x^2 + 6x + 15\ \ R\ 34$, solve for a. Show your work.

$(2)^4+2(2)^2+a(2)+4=34$

$16 + 8 + 2a + 4 = 34$

$\cancel{28} + 2a = 34$

$\cancel{-28} \quad -28$

$2a = 6$

$\boxed{a=3}$

OR

$$\begin{array}{r|rrrrr} 2 & 1 & 0 & 2 & a & 4 \\ & \downarrow & 2 & 4 & 12 & 30 \\ \hline & 1 & 2 & 6 & 15 & \boxed{34} \end{array}$$

$\boxed{a=3}$

I can divide polynomials and factor completely. (C15)

20. Given $x + 3$ is a factor of $c(x)$, factor $c(x) = x^3 + 7x^2 - 36$ completely.

$$\begin{array}{r|rrrr} -3 & 1 & 7 & 0 & -36 \\ & \downarrow & -3 & -12 & 36 \\ \hline & 1 & 4 & -12 & \boxed{0} \end{array}$$

$x^2+4x-12$

-12
6 -2
4

$(x+6)(x-2)$

$\boxed{(x+3)(x+6)(x-2)}$

*don't forget original factor of $(x+3)$!

I can factor higher order polynomials using grouping and by using quadratic form. (C16)

21. Factor completely. $x^3 - 3x^2 - 16x + 48$

$\underbrace{x^3-3x^2}\ \underbrace{-16x+48}$

$x^2(x-3)-16(x-3)$

$(x^2-16)(x-3)$

diff. of sq.

$\boxed{(x+4)(x-4)(x-3)}$

22. Factor completely. $x^4 - 40x^2 + 144$

144, −4, −36, −40

$(x^2-4)(x^2-36)$

both are diff. of squares

$\boxed{(x+2)(x-2)(x+6)(x-6)}$

I can factor a sum of cubes and a difference of cubes. (C17)

23. Write in factored form: $y^3 - 64$

difference of cubes

$\boxed{(y-4)(y^2+4y+16)}$

24. Write in factored form: $8x^3 + 27$

sum of cubes

$\boxed{(2x+3)(4x^2-6x+9)}$

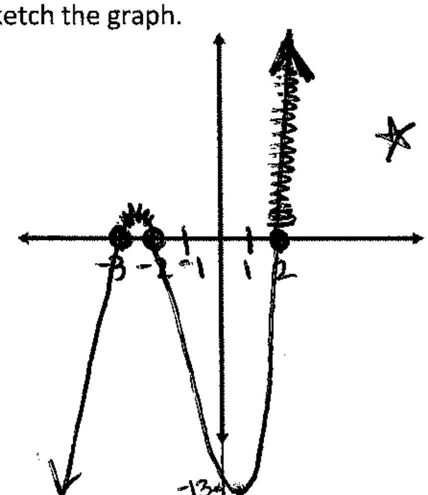
I can find the solution to a polynomial inequality graphically and algebraically. (C18)

25. Solve by factoring and sketching:
$x^3 + 3x^2 - 4x - 12 > 0$

Zoom 6

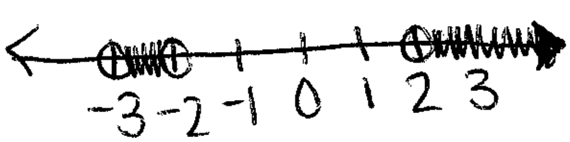
x-int. at $(-3,0)$, $(-2,0)$, & $(2,0)$

a. Sketch the graph.



★ look at parts of graph above the x-axis (greater than y-value of zero)

b. Draw a number line to represent your solution.



★ open circles on x-values

c. Write your solution in interval notation.

$(-3,-2) \cup (2,\infty)$

I can find the solution to a polynomial inequality graphically. I can represent a solution to a polynomial inequality using inequalities, interval notation, number lines, and verbal descriptions in context. C19

Emilio has been trying to regulate the pH level in his aquarium for 5 hours. For the water to be safe for the fish, he must keep the pH less than or equal to 9. The function $p(x) = -0.34x^3 + 2.652x^2 - 5.4638x + 11.1114$ *represents the pH level in the tank* x *hours since Emilio began to regulate it.*

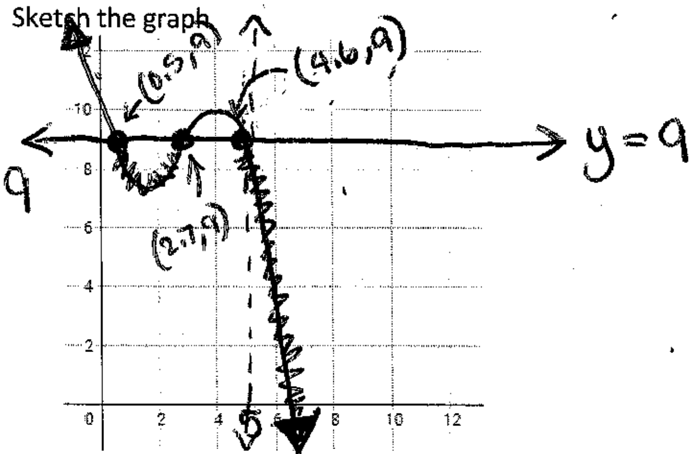
26. Write and solve an inequality to determine the time intervals during which the pH was safe for Emilio's fish. State your solution in interval notation. Explain in words what your solution means. Sketch the graph.

a. Write and solve an inequality to determine the time intervals during which the pH was safe for Emilio's fish.

$-0.34x^3 + 2.652x^2 - 5.4638x + 11.1114 \leq 9$

Solution: $0.5 \leq x \leq 2.7$
(approx) or $4.6 \leq x \leq 5$

b. Sketch the graph.



c. State your solution in interval notation.

$[0.5, 2.7] \cup [4.6, 5]$

regulating for 5 hours

d. Explain, in words, what your solution means.

Emilio's aquarium's pH value is safe for fish between 0.5 & 2.7 hours (inclusive) and from 4.6 hours up to the 5th hour that he was trying to regulate.