Math 3 – Standard Form of a Circle Name ______________________________

In order to put the equation of a circle into standard form, you must complete the square just like you do with a quadratic equation.

PART A: Let's review completing the square with quadratics first!

1. In order to complete the square, you must make the quadratic into a perfect square trinomial. Remember: a perfect square trinomial has a c-value that is equal to $\left(\frac{b}{2}\right)^2$. For the problems below, determine the c value that would make the quadratic a perfect square trinomial.

a) $x^2 + 8x +$ ____ b) $x^2 + 4x +$ ____ c) $x^2 - 10x +$ ____

2. Once you have found the perfect square trinomial, you must factor the quadratic. You can always use the diamond but the shortcut for factoring a perfect square trinomial is $\left(x + \frac{b}{2}\right)^2$. For the perfect square trinomials you found above, put the quadratics into factored form using the shortcut.

a) b) c)

PART B: To identify the center and the radius of a circle written in general form, it is necessary to rewrite the equation in standard form. We will use completing the square for this process.

Here is an example: Rewrite $x^2 + y^2 - 4x - 6y + 9 = 0$ in standard form by completing the square.

Step 1: Move the constant term to the other side.	$x^2 + y^2 - 4x - 6y = -9$
Step 2: Group the x-terms together and the y-terms together using sets of parentheses.	$(x^2 - 4x) + (y^2 - 6y) = -9$
Step 3: Complete the square within each parenthesis by creating a perfect square trinomial and add the constant term to both sides.	$(x^2 - 4x + 4) + (y^2 - 6y + 9) = -9 + 4 + 9$
Step 4: Factor the left side of the equation and simplify the right side.	$(x - 2)^2 + (y - 3)^2 = 4$

Now your equation is in general form and you can identify the center and radius of the circle!

Center: Radius:

Practice: Rewrite each equation in general form and identify the center and radius of the circle.

3. $x^2 + y^2 - 2x + 4y + 4 = 0$

4. $x^2 + y^2 - 10x + 12y + 51 = 0$

PART C: Consider the general form of the equation $2x^2 + 2y^2 - x + 4y + 2 = 0$.

5. How do the coefficients of the squared terms in this equation differ from the equation in the example?

6. How will you change your strategy or adjust your process for rewriting this equation in standard form?

7. Rewrite $2x^2 + 2y^2 - x + 4y + 2 = 0$ in standard form.

Step 1: Divide every term by the squared terms coefficient so both square terms have a coefficient of 1.	
Step 2: Move the constant term to the other side.	
Step 3: Group the x-terms together and the y-terms together using sets of parentheses.	
Step 4: Complete the square within each parenthesis by creating a perfect square trinomial and add the constant term to both sides.	
Step 5: Factor the left side of the equation and simplify the right side.	

Practice: Rewrite each equation in general form and identify the center and radius of the circle.

8. $2x^2 + 2y^2 - 5x + 8y + 10 = 0$.

Consider the general form of the equation of $x^2 + 2y^2 - x + 10y + 25 = 0$. This equation does NOT represent a circle.

9. What is different about this equation than any of the other equations that represent a circle?

10. How can you tell that an equation is not a circle?