

Factor each expression completely.

1)  $x^3 - 6x^2 - 9x + 54$

1. First, I checked to see if there was a GCF - there isn't. Then I counted the number of terms and saw that there were four terms. The only way we've learned to factor four terms is with grouping, so that is what I did.
2. I created two groups, then looked for a GCF for each group.
3. In the first group, both terms have  $x^2$  as a factor. In the second group, both terms have 9 as a factor. But when I took out 9, my remaining groups didn't match, so I went back and took  $-9$  from the second group.
4. Once the GCFs were out and I saw that my two sets of parentheses were the same, I just treated the  $(x - 6)$  like another GCF and factored that out, leaving me with  $(x^2 - 9)$ .
5. I recognized that  $(x^2 - 9)$  was the difference of two squares.  $x^2$  is obviously a square and 9 is the same as  $3^2$ , so I factored it further using the short-cut for factoring a difference of squares.

1.  $x^3 - 6x^2 - 9x + 54$
2.  $(x^3 - 6x^2) + (-9x + 54)$
3.  $x^2(x - 6) - 9(x - 6)$
4.  $(x - 6)(x^2 - 9)$
5.  $(x - 6)(x + 3)(x - 3)$

2)  $x^4 - 29x^2 + 100$

1. First I checked to see if there was a GCF - there isn't. Then I counted the number of terms and saw that there were three terms with exponents that count down evenly. This is similar to a quadratic trinomial, even though the first term is degree four. The exponents here are counting down by twos.  
Next I checked to see if my leading coefficient (my a-value) was equal to 1 or not. It was equal to 1 so I thought I could use the diamond method short-cut like I would do if it was a quadratic trinomial.
2. Since I have  $x^4$  instead of  $x^2$ , I made a substitution to make it easier for me to put into the diamond. I let  $z = x^2$  which means that  $z^2 = x^4$ . That way I could find my factors lickety-split.
3. After I found  $-25$  and  $-4$  using a diamond, I just put them into my factors with  $z$  because  $a=1$ .
4. Then I went back and replaced  $z$  with  $x^2$ .
5. I now had two factors that were both the difference of two squares ( $25 = 5^2$  and  $4 = 2^2$ ). I factored those using the short-cut for factoring a difference of squares into the four factors I ended with.

1.  $x^4 - 29x^2 + 100$
2.  $z^2 - 29z + 100$   
 $a = 1$   
 $b = -29$   
 $c = 100$
3.  $(z - 25)(z - 4)$
4.  $(x^2 - 25)(x^2 - 4)$
5.  $(x + 5)(x - 5)(x + 2)(x - 2)$

3)  $x^3 + 27$

- First I checked to see if there was a GCF - there isn't. Then I counted the number of terms and saw that there were two terms. Next I checked to see if I had addition or subtraction. Since I had addition there was only one thing for me to try - the sum of two cubes.  $x^3$  is obviously a perfect cube and I know that  $27 = 3^3$ , so I used the pattern we learned to factor the sum of cubes. Remember:  $a^3 + b^3 = (a + b)(a^2 - ab + b^2)$

$$\begin{aligned} & x^3 + 27 \\ & (x)^3 + (3)^3 \\ & (x + 3)(x^2 - (x)(3) + 3^2) \\ & (x + 3)(x^2 - 3x + 9) \end{aligned}$$

## Factoring Decision Tree:



## Additional Practice:

1. Factor the **GCF** from each of these polynomials.

a)  $x^4 + 64x$

b)  $24x^3 + 42x^2 - 12x$

c)  $2x^2 - 50$

d)  $x^5 - 10x^3 + 9x$

2. Use the **factoring decision tree above** to help you determine which additional type of factoring you should try with each part of question one. Write what method you should try for each.

a)

b)

c)

d)

3. Now factor each of the expressions from #1 **COMPLETELY** using their GCFs and the type of factoring method that you chose for each one in #2. Box/circle each one of your answers.

a)  $x^4 + 64x$

b)  $24x^3 + 42x^2 - 12x$

c)  $2x^2 - 50$

d)  $x^5 - 10x^3 + 9x$