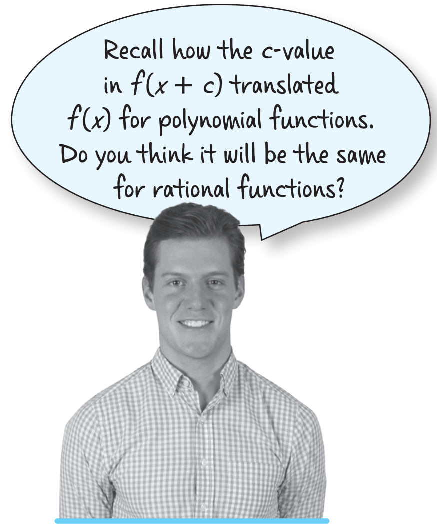
PROBLEM 1 Shifty Behavior, Take 1

Recall from *A Rational Existence* that the reciprocal of power functions have a vertical asymptote at $x = 0$ and a horizontal asymptote at $y = 0$. The domain is all real numbers except for 0, because division by 0 is undefined.

In this problem you will use a graphing calculator to explore rational functions of the form

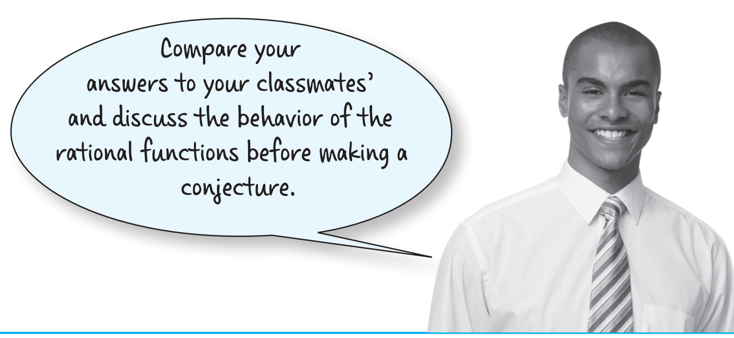
$g(x) = \frac{1}{x - c}$ for a constant value c.



1. Consider the table shown.

a. Identify the vertical asymptote, horizontal asymptote, domain, and range for the given *c*-values. Then choose different positive and negative *c*-values to complete the table.

c-value	$g(x) = \frac{1}{x - c}$	Vertical Asymptote(s)	Horizontal Asymptote(s)	Domain	Range
1	$g(x) = \frac{1}{x - 1}$				
−2	$g(x) = \frac{1}{x + 2}$				



b. Determine the general formula to identify the vertical asymptote of a rational function in the form $g(x) = \frac{1}{x - c}$. Explain your reasoning.

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c. What generalization(s) can you make about the *c*-value and the domain? The range?

d. What effect does changing the *c*-value have on the function's end behavior? Explain your reasoning.

2. Without using a graphing calculator, determine the domain, range, and vertical and horizontal asymptotes of each rational function.

a. $f(x) = \frac{10}{x}$

Domain:

Range:

Vertical Asymptote:

Horizontal Asymptote:

b. $g(x) = \frac{1}{x + 10}$

Domain:

Range:

Vertical Asymptote:

Horizontal Asymptote:

c. $j(x) = 10x$

Domain:

Range:

Vertical Asymptote:

Horizontal Asymptote:

d. $g(x) = \frac{1}{x - 10}$

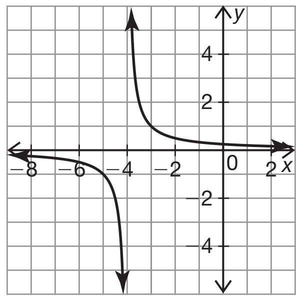
Domain:

Range:

Vertical Asymptote:

Horizontal Asymptote:

3. Write the rational function(s) from the graph, table, or description provided. Explain your reasoning.

a. 

Function: ________________

Explanation:

b. Vertical asymptote at $x = 3$ and a horizontal asymptote at $y = 0$.

Function 1: ________________

Function 2: ________________

Explanation:

c. Domain: All Real Numbers except $x = 7$
Range: All Real Numbers except $y = 0$

Function 1: ________________

Function 2: ________________

Explanation:

PROBLEM 2 Ctrl-Alt-Shift

Consider the functions $y = f(x)$ and $g(x) = Af(B(x - C)) + D$. Recall that adding a constant D translates $f(x)$ vertically, while adding a constant C translates $f(x)$ horizontally. Multiplying by the constant A dilates $f(x)$ vertically, while multiplying by the constant B dilates $f(x)$ horizontally. Rational functions are transformed in the same manner.

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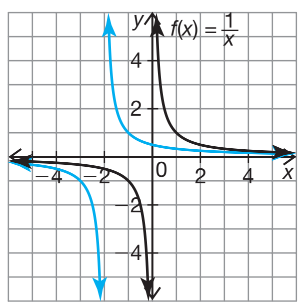
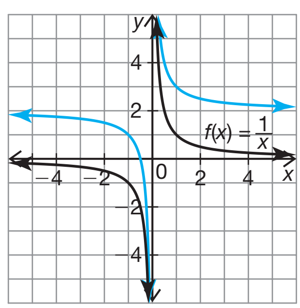
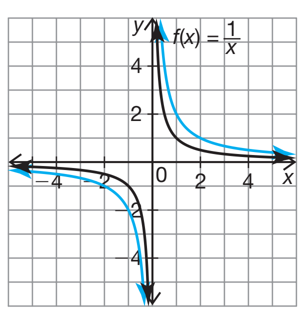
1. The function $f(x) = \frac{1}{x}$ is shown in black on each coordinate plane. Determine whether the second function on each graph is $j(x) = \frac{1}{x + 2}$, $m(x) = \frac{2}{x}$, or $k(x) = \frac{1}{x} + 2$. Explain your reasoning.

	Function: Explanation:
	Function: Explanation:
	Function: Explanation:

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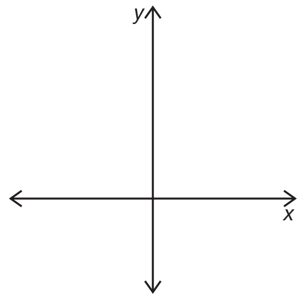
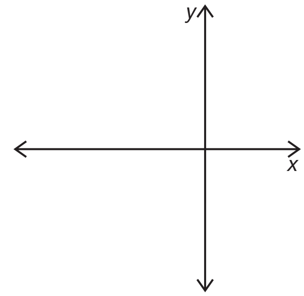
2. Given $f(x) = \frac{1}{x}$.

a. Sketch $g(x) = f(x) + 5$



Explanation:

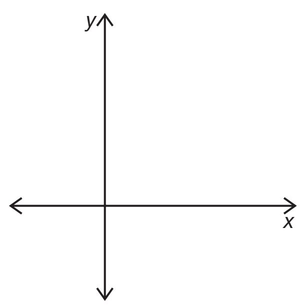
b. Sketch $h(x) = f(x + 5)$.



Explanation:

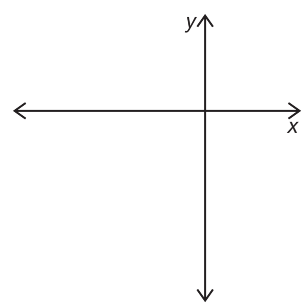
3. Write a rational function $g(x)$ that matches the given characteristics. Sketch the function on the coordinate plane.

a. Vertical asymptote at $x = 2$
Horizontal asymptote at $y = 1$



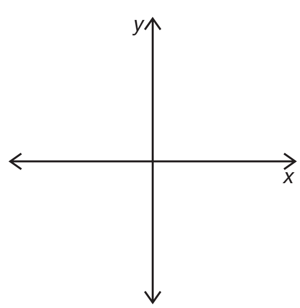
$g(x) =$

b. Vertical asymptote at $x = 1, x = -5$
Horizontal asymptote at $y = -3$



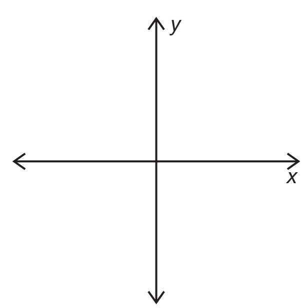
$g(x) =$

c. For $f(x) = \frac{1}{x}$, $g(x) = f(x - 2) - 4$



$g(x) =$

d. For $f(x) = \frac{1}{x}$, $g(x)$ shifts $f(x)$ up and to the left.



$g(x) =$

Be prepared to share your solutions and methods.