PROBLEM 3 Shifty Behavior, Take 3

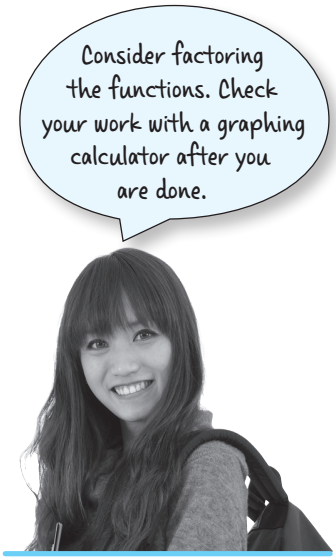
In the previous problems in this chapter, you analyzed rational functions with just 1 vertical asymptote. The vertical asymptote occurred at the value for which the denominator was zero.

1. Without graphing, determine the number of vertical asymptotes for each function. Show all work and explain your reasoning.

a. $f(x) = \frac{4}{x^2 + 4}$

b. $g(x) = \frac{4}{x^2 - 4}$

c. $h(x) = \frac{4}{x^2 + 4x + 4}$



Recall from the Fundamental Theorem of Algebra that a function of degree n has n zeros. Some of the zeros may be imaginary. Therefore, it follows that the reciprocal of a function of degree n can have at most n vertical asymptotes.

2. Sarah determines the vertical asymptotes for the function $f(x) = \frac{1}{2x^2 - 14x - 16}$.

Sarah

The terms in the denominator have a common factor of 2, so I factored it out first. Then I factored the remaining quadratic.

$$f(x) = \frac{1}{2(x^2 - 7x - 8)} = \frac{1}{2(x - 8)(x + 1)}$$

Vertical asymptotes occur when the denominator is zero. So, the asymptotes will occur when $x - 8 = 0$ and when $x + 1 = 0$.

Therefore, the asymptotes occur at $x = 8$ and $x = -1$.

Is Sarah correct? Explain your reasoning.

3. Analyze each rational function. Use algebra to determine the vertical asymptotes.

a. $f(x) = \frac{5}{7x - 35}$

b. $g(x) = \frac{1}{x(x - 2)(2x + 3)}$

c. $h(x) = \frac{10}{x^2 - 3x - 10}$

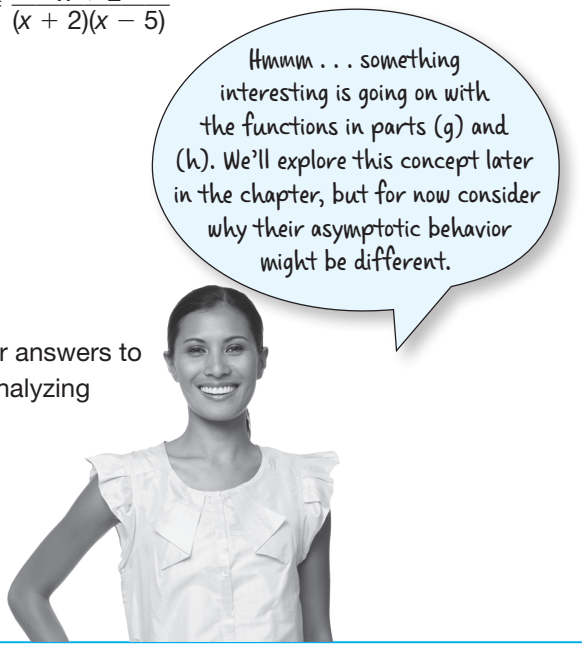
d. $h(x) = \frac{x}{2x^2 + 9x + 4}$

e. $h(x) = \frac{7}{x^4 - 1}$

f. $f(x) = \frac{2}{x^2 + 2}$

g. $h(x) = \frac{x - 2}{x - 2}$

h. $g(x) = \frac{x + 2}{(x + 2)(x - 5)}$



i. Use a graphing calculator to check your answers to Questions 3 by graphing and then by analyzing the table of values.

4. Determine 2 different rational functions with the characteristics given.

a. vertical asymptotes at $x = 3$, $x = -1$, and $x = 0$

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b. vertical asymptotes at $x = \frac{1}{2}$ and $x = 2$

Be prepared to share your solutions and methods.