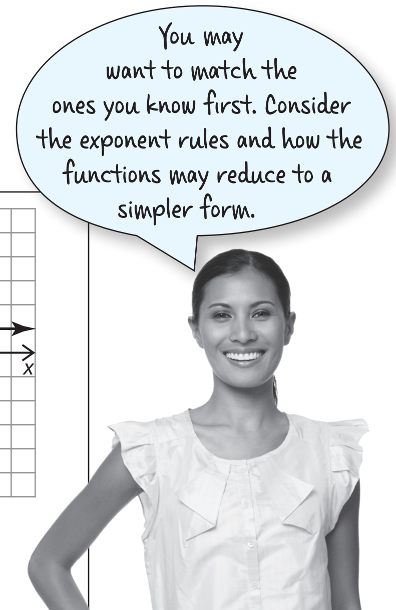
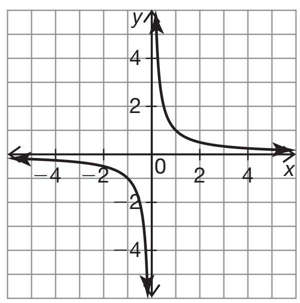
PROBLEM 1 Mend Your Function, Dear Henry!

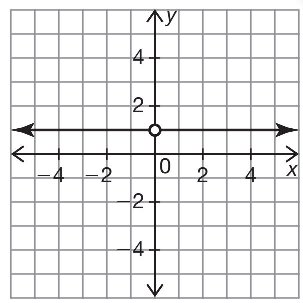
1. Without using a graphing calculator, match each rational function provided with the correct graph. Write the function below the graph.

$y = \frac{1}{x - 2}$	$y = \frac{1}{x}$	$y = \frac{x^2}{x}$	$y = \frac{x - 2}{x - 2}$
$y = \frac{x^3}{x}$	$y = \frac{(x - 2)^2}{x - 2}$	$y = \frac{x}{x}$	$y = \frac{(x - 2)^3}{x - 2}$

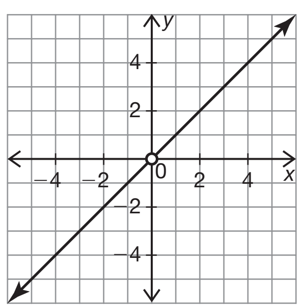




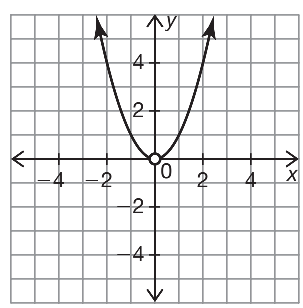
Function:



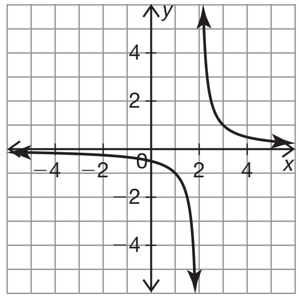
Function:



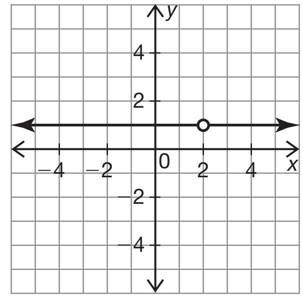
Function:



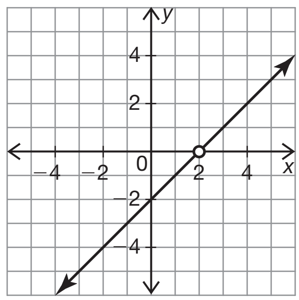
Function:



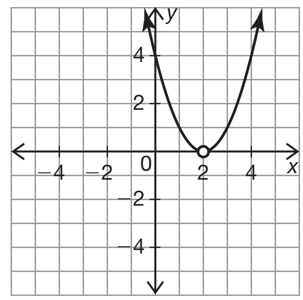
Function:



Function:



Function:



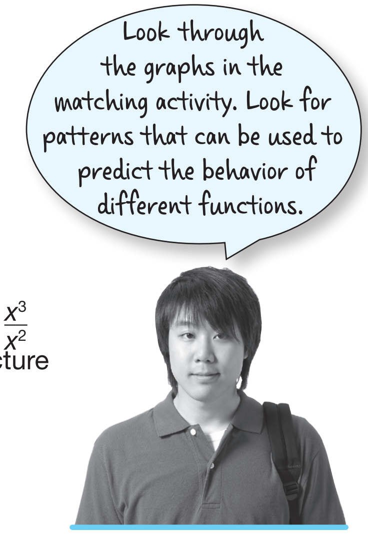
Function:

9

a. Which functions have asymptotes and which functions have "holes" in their graphs? Describe how the structure of the equation determines whether the function will have an asymptote or a "hole."

b. Compare the graphs of $y = \frac{1}{x - 2}$ and $y = \frac{x - 2}{x - 2}$. How are they the same? How are they different? Describe how the structure of the equation determines these differences.

c. Compare the graphs of $y = \frac{x}{x}$ and $y = \frac{x-2}{x-2}$. How are they the same? How are they different? Describe the similarities and differences in the domain and range in terms of the structure of their equations.

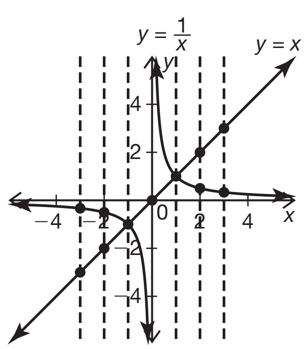


d. Without using a graphing calculator, describe the similarities and differences between the graphs of $y = \frac{x^3}{x^2}$ and $y = x$. Explain your reasoning in terms of the structure of the equations.

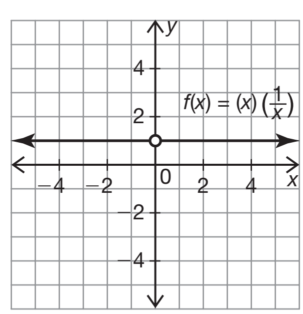
Many rational functions have "holes," or breaks, in the graphs instead of asymptotes. Let's analyze the structure of the function $y = \frac{x}{x}$ to determine why this function has a "hole" in the graph rather than a vertical asymptote at $x = 0$.

The function $y = \frac{x}{x}$ can be rewritten as the product of two factors: $y = (x)\left(\frac{1}{x}\right)$. Looking at these reciprocal factors as separate functions reveals important characteristics.

x	-4	-3	-2	-1	0	1	2	3
$y = (x)\left(\frac{1}{x}\right)$	$(-4)\left(-\frac{1}{4}\right)$ 1	$(-3)\left(-\frac{1}{3}\right)$ 1	$(-2)\left(-\frac{1}{2}\right)$ 1	$(-1)\left(-\frac{1}{1}\right)$ 1	$(0)\left(\frac{1}{0}\right)$ und	$(1)\left(\frac{1}{1}\right)$ 1	$(2)\left(\frac{1}{2}\right)$ 1	$(3)\left(\frac{1}{3}\right)$ 1



Graphical representation of each factor



Graphical representation of the product

PROBLEM 2 With What Shall I Mend The Function, Dear Liza?

1. Henry graphed the rational function $f(x) = \frac{x^2 + x - 6}{x - 2}$. Analyze his work.

Henry

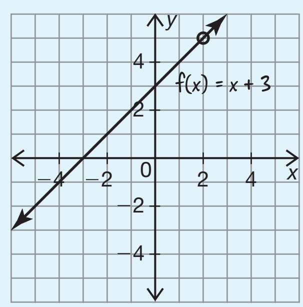
I know there is a domain restriction, so $x \neq 2$. I'm not sure if this is a vertical asymptote or a removable discontinuity, so I'm going to factor the numerator, if possible, to see if a common factor exists.

$$f(x) = \frac{x^2 + x - 6}{x - 2} = \frac{(x - 2)(x + 3)}{x - 2}$$

$$= \frac{1(x + 3)}{1} = x + 3$$

I know the output values of $\frac{(x - 2)}{(x - 2)} = 1$ with a discontinuity at $x = 2$. Therefore I can simply graph $f(x) = x + 3$. The removable discontinuity is at $(2, 2 + 3)$ and appears as a "hole" in the graph.

Note that the common factors do not "cancel." Many people use this term incorrectly to describe when factors divide to equal 1.



a. Why did Henry include an open circle at (2, 5) and *not* a vertical asymptote at $x = 2$?

b. Explain why $f(x) = \frac{x^2 + x - 6}{x - 2}$ can be rewritten as $f(x) = x + 3$.

The graphs of rational functions will have either a removable discontinuity or a vertical asymptote for all domain values that result in division by 0. Simplifying rational expressions is similar to simplifying rational numbers: common factors divide to 1.

2. Analyze the table that shows similarities between rational numbers and rational functions.

		Rational Numbers	Rational Expressions
A common numerator and denominator divide to equal 1.	**Examples**	$\frac{5}{5} = 1$	$\frac{x}{x} = 1$
		$\frac{10.7}{10.7} = 1$	$\frac{5x}{5x} = 1$
		$\frac{0.025 + 0.016}{0.025 + 0.016} = 1$	$\frac{x + 5}{x + 5} = 1$
Common monomial factors divide to equal 1.	**Examples**	$\frac{5 \cdot 3}{5} = \frac{1 \cdot 3}{1} = 3$	$\frac{5x}{5} = \frac{1 \cdot x}{1} = x$
		$\frac{4}{4 \cdot 6} = \frac{1}{1 \cdot 6} = \frac{1}{6}$	$\frac{x}{xz} = \frac{1}{1 \cdot z} = \frac{1}{z}$
Common binomial factors divide to equal 1.	**Examples**	$\frac{(5 + 3)(16 - 7)}{(5 + 3)} = \frac{1 \cdot (16 - 7)}{1} = 16 - 7$	$\frac{(x + 5)(x - 4)}{(x + 5)} = \frac{1(x - 4)}{1} = (x - 4)$
		$\frac{(9 - 4)}{(9 - 4)(9 + 5)} = \frac{1}{(9 + 5)}$	$\frac{x - 4}{(x - 4)(x + 5)} = \frac{1}{(x + 5)}$

a. Describe how simplifying rational numbers is similar to simplifying rational expressions.

b. Why is there a 1 in the numerator after simplifying $\frac{x}{xz} = \frac{1}{z}$?

c. For each example in the rational expressions column, list any restrictions on the domain.

3. Liza rewrites the rational expression as shown.

Liza

$$\frac{x^2 + 4x + 3}{4x + 3} = x^2$$

I divided out the common factors. The numerator and denominator each have a 4x and a 3, so I am left with the squared term.

Describe the error in Liza's reasoning.

4. Simplify each rational expression. List any restrictions on the domain.

a. $\frac{2x^2 - 8}{x - 2}$

b. $\frac{3xy - 3y}{x^2 - 1}$

c. $\frac{x^2 - 5x + 6}{3x - 9}$

d. $\frac{x^3 - 7x^2 - 18x}{3x^2 - 9x}$

e. $\frac{25x^2 - 9}{5x^2 - 12x - 9}$

f. $\frac{x^3 - 5x^2 - x + 5}{x^2 - 6x + 5}$

PROBLEM 3 Use Your Head, Dear Henry!

1. Determine whether the graph of the rational function has a vertical asymptote, a removable discontinuity, both, or neither. List the discontinuities and justify your reasoning.

   a. $j(x) = \dfrac{x + 2}{x(x + 2)}$

   b. $h(x) = \dfrac{x}{x + 5}$

   c. $j(x) = \dfrac{5}{5(x + 2)}$

   d. $j(x) = \dfrac{x + 2}{x^2 - 2x - 15}$

2. Write two examples of rational functions with one or more removable discontinuities. Explain your reasoning.

3. Write a unique function that has a vertical asymptote and a removable discontinuity. Explain your reasoning.