PROBLEM 3 Use Your Head, Dear Henry!

1. Determine whether the graph of the rational function has a vertical asymptote, a removable discontinuity, both, or neither. List the discontinuities and justify your reasoning.

   a. $j(x) = \dfrac{x + 2}{x(x + 2)}$

   b. $h(x) = \dfrac{x}{x + 5}$

   c. $j(x) = \dfrac{5}{5(x + 2)}$

   d. $j(x) = \dfrac{x + 2}{x^2 - 2x - 15}$

2. Write two examples of rational functions with one or more removable discontinuities. Explain your reasoning.

3. Write a unique function that has a vertical asymptote and a removable discontinuity. Explain your reasoning.

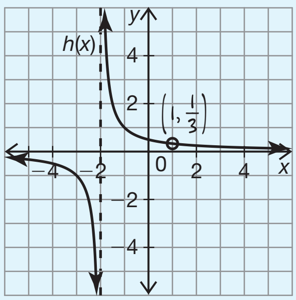
4. Liza graphed the rational function $h(x) = \frac{x - 1}{x^2 + x - 2}$. Analyze her work.

Liza

I'm not sure where the asymptotes are, so I'm going to factor the denominator if possible.

$$h(x) = \frac{x - 1}{x^2 + x - 2} = \frac{x - 1}{(x - 1)(x + 2)}$$
$$= \frac{1}{x + 2}.$$

I know there are domain restrictions at $x = 1$ and $x = -2$. The common factor $(x - 1)$ is in the numerator so $\frac{x - 1}{x - 1} = 1$. Therefore $x = 1$ is a removable discontinuity, while $x = -2$ is a vertical asymptote. I can quickly sketch $h(x) = \frac{1}{x + 2}$ as a horizontal shift of $h(x) = \frac{1}{x}$ two units to the left. I know a discontinuity will exist at $\left(1, \frac{1}{1 + 2}\right)$, or $\left(1, \frac{1}{3}\right)$. A horizontal asymptote is at $y = 0$ and the y-intercept is $\left(0, \frac{1}{2}\right)$.

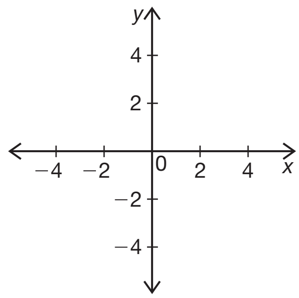


a. Summarize why $x = -2$ is a vertical asymptote while $x = 1$ appears as a "hole" in the graph.

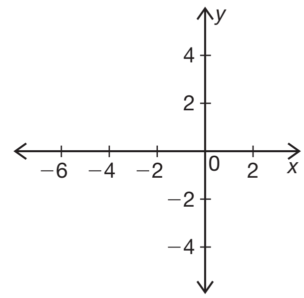
b. Explain why the graph has a horizontal asymptote at $y = 0$.

5. Sketch each function without the use of a graphing calculator. Identify any restrictions.

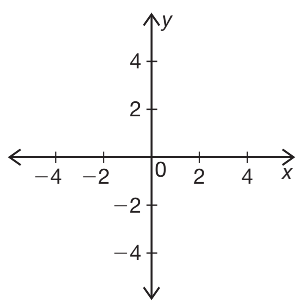
a. $f(x) = \dfrac{x + 2}{x^2 + 4x + 4}$



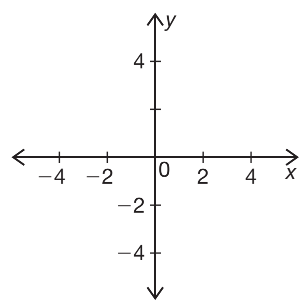
b. $g(x) = \dfrac{x}{x^2 + 3x}$



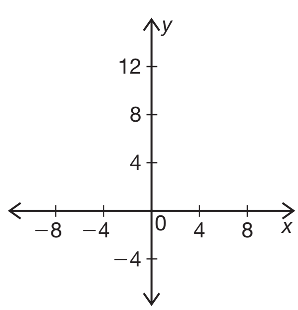
c. $h(x) = \dfrac{x}{x^3 - x}$



d. $m(x) = \dfrac{x^2 + 5x + 6}{x^2 + 5x + 6}$



e. $k(x) = \dfrac{x^2 - 2x - 15}{x - 5}$



f. $k(x) = \dfrac{x^3 + x^2 + 2x + 2}{x + 1}$