PROBLEM 1 Oh Snap . . . Look at the Denominator on that Rational

Previously, you learned that dividing polynomials was just like dividing integers. Well, performing operations on rational expressions involving variables is just like performing operations on rational numbers.

	Rational Numbers	Rational Expressions Involving Variables in the Numerator
Example 1	$\frac{1}{6} + \frac{5}{6} - \frac{1}{6} = \frac{5}{6}$	$\frac{1x}{6} + \frac{5x}{6} - \frac{1x}{6} = \frac{5x}{6}$
Example 2	$\frac{3}{2} + \frac{2}{5} - \frac{3}{4} = \frac{3(10)}{2(10)} + \frac{2(4)}{5(4)} - \frac{3(5)}{4(5)}$ $= \frac{30}{20} + \frac{8}{20} - \frac{15}{20}$ $= \frac{23}{20}$	$\frac{3x}{2} + \frac{2y}{5} - \frac{3x}{4} = \frac{3(10)x}{2(10)} + \frac{2(4)y}{5(4)} - \frac{3(5)x}{4(5)}$ $= \frac{30x}{20} + \frac{8y}{20} - \frac{15x}{20}$ $= \frac{15x + 8y}{20}$
Example 3	$\frac{3}{5} + \frac{2}{3} - \frac{2}{15} = \frac{3(3)}{5(3)} + \frac{2(5)}{3(5)} - \frac{2}{15}$ $= \frac{9}{15} + \frac{10}{15} - \frac{2}{15}$ $= \frac{17}{15}$	$\frac{3x}{5} + \frac{2y}{3} - \frac{2}{15} - \frac{2x + 3y}{5} = \frac{3(3)x}{5(3)} + \frac{2(5)y}{3(5)} - \frac{2}{15} - \frac{(2x + 3y)(3)}{5(3)}$ $= \frac{9x}{15} + \frac{10y}{15} - \frac{2}{15} - \frac{6x + 9y}{15}$ $= \frac{9x + 10y - 2 - (6x + 9y)}{15}$ $= \frac{9x + 10y - 2 - 6x - 9y}{15}$ $= \frac{3x + y - 2}{15}$

1. Analyze the examples.

   a. Explain the process used to add and subtract each expression.

   b. In Example 2, why is $\frac{3}{2} = \frac{3(10)}{2(10)}$ and why is $\frac{3x}{2} = \frac{3(10)x}{2(10)}$?

   c. In Example 3, explain how $-\frac{(2x + 3y)(3)}{5(3)} = \frac{-6x - 9y}{15}$.

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2. Analyze Noelle's work.

> Noelle
>
> $$\frac{3x}{3} + \frac{2x}{8} - \frac{1}{2}$$
>
> To determine a common denominator, I multiply all the denominators together: $3 \cdot 8 \cdot 2 = 48$
>
> $$\frac{3x(16)}{3(16)} + \frac{2x(6)}{8(6)} - \frac{1(24)}{2(24)} = \frac{48x}{48} + \frac{12x}{48} - \frac{24}{48}$$
>
> $$= \frac{60x - 24}{48}$$
>
> $$= \frac{5x - 2}{4}$$

Explain how Noelle could have added the rational expressions more efficiently.

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3. Calculate each sum and difference.

a. $\frac{3}{6} + \frac{5x}{4} - \frac{y}{8}$

b. $\frac{x - 2y}{3} + \frac{x}{12} - \frac{z}{4}$

c. $\frac{4x}{6} - \frac{2x}{9} - \frac{x}{18}$

4. Is the set of rational expressions closed under addition and subtraction? Explain your reasoning.

5. Notice that all the variables in the right column of the table are in the numerator. If there were variables in the denominator, do you think the process of adding and subtracting the expressions would change? Explain your reasoning

PROBLEM 2 Umm, I Think There Are Some Variables in Your Denominator . . .

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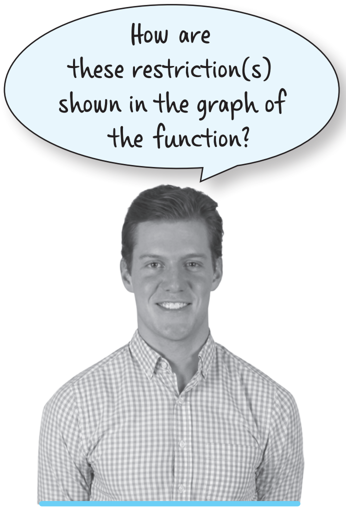
When rational expressions contain variables in the denominator, the process remains the same—you still need to determine the least common denominator (LCD) before adding and subtracting.

It will save time and effort if you determine the LCD and use it to add and subtract rational expressions.

1. Consider Method A compared to Method B in both columns of the table.

	Rational Numbers	Rational Expressions Involving Variables in the Denominator
Method A	$\frac{1}{3} + \frac{1}{3^2} = \frac{1(3^2)}{3(3^2)} + \frac{1(3)}{3^2(3)}$ $= \frac{3^2}{3^3} + \frac{3}{3^3}$ $= \frac{3^2 + 3}{3^3}$ $= \frac{3(3 + 1)}{3(3^2)}$ $= \frac{3 + 1}{3^2}$	$\frac{1}{x} + \frac{1}{x^2} = \frac{1(x^2)}{x(x^2)} + \frac{1(x)}{x^2(x)}$ $= \frac{x^2}{x^3} + \frac{x}{x^3}$ $= \frac{x^2 + x}{x^3}$ $= \frac{x(x + 1)}{x(x^2)}$ $= \frac{x + 1}{x^2}$ $\frac{1}{x} + \frac{1}{x^2} = \frac{x + 1}{x^2}$ for $x \neq 0$
Method B	$\frac{1}{3} + \frac{1}{3^2} = \frac{1(3)}{3(3)} + \frac{1}{3^2} = \frac{4}{3^2}$	$\frac{1}{x} + \frac{1}{x^2} = \frac{1(x)}{x(x)} + \frac{1}{x^2}$ $= \frac{x + 1}{x^2}$ $\frac{1}{x} + \frac{1}{x^2} = \frac{x + 1}{x^2}$ for $x \neq 0$

a. Explain the difference in the methods.



b. Explain why the statement $\frac{1}{x} + \frac{1}{x^2} = \frac{x + 1}{x^2}$ has the restriction $x \neq 0$.

2. Ruth and Samir determine the LCD for the expression: $\frac{1}{x^2 - 1} + \frac{1}{x + 1}$.

Ruth

$\frac{1}{x^2 - 1} + \frac{1}{x + 1}$

$(x^2 - 1)(x + 1)$

LCD: $x^3 + x^2 - x - 1$

Samir

$\frac{1}{x^2 - 1} + \frac{1}{x + 1}$

$\frac{1}{(x - 1)(x + 1)} + \frac{1}{x + 1}$

$(x - 1)(x + 1)$

LCD: $x^2 - 1$

a. Who is correct? Explain your reasoning.

b. Describe any restriction(s) for the value of x.

3. Calculate the least common denominator for each pair of rational expressions.

a. $\dfrac{3}{x + 4}, \dfrac{7x}{x - 4}$

LCD:

b. $\dfrac{-2}{3x - 2}, \dfrac{4x}{3x^2 + 7x - 6}$

LCD:

c. $\dfrac{-11}{x}, \dfrac{7}{x - 4}, \dfrac{x}{x^2 - 16}$

LCD:

d. $\dfrac{2x}{x^2 - 5x + 6}, \dfrac{7x + 11}{x^2 - 6x + 9}$

LCD:

Notice that even though there are binomials in the denominator, adding two rational expressions is similar to adding two rational numbers.

Rational Expressions Involving Binomials in the Denominator

$$\frac{1}{x^2 - 1} - \frac{1}{x + 1} = \frac{1}{(x + 1)(x - 1)} - \frac{1}{x + 1}$$

$$= \frac{1}{(x + 1)(x - 1)} - \frac{1(x - 1)}{(x + 1)(x - 1)}$$

$$= \frac{1}{(x + 1)(x - 1)} - \frac{x - 1}{(x + 1)(x - 1)}$$

$$= \frac{1 - x + 1}{(x + 1)(x - 1)}$$

$$= \frac{-x + 2}{(x + 1)(x - 1)}$$

4. Marissa and Salvatore add $\frac{2x + 2}{x + 1} + \frac{1}{x}$.

Marissa

$$\frac{2x + 2}{x + 1} + \frac{1}{x} = \frac{(2x + 2)(x)}{(x + 1)(x)} + \frac{1(x + 1)}{x(x + 1)}$$
$$= \frac{2x^2 + 2x}{(x + 1)(x)} + \frac{x + 1}{x(x + 1)}$$
$$= \frac{2x^2 + 2x + x + 1}{x(x + 1)}$$
$$= \frac{2x^2 + 3x + 1}{x(x + 1)}$$
$$= \frac{(2x + 1)\cancel{(x + 1)}}{x\cancel{(x + 1)}}$$
$$= \frac{2x + 1}{x}$$

Salvatore

$$\frac{2x + 2}{x + 1} + \frac{1}{x} = \frac{2\cancel{(x + 1)}}{\cancel{(x + 1)}} + \frac{1}{x}$$
$$= 2 + \frac{1}{x}$$
$$= \frac{2(x)}{(x)} + \frac{1}{x}$$
$$= \frac{2x + 1}{x}$$

Explain the difference in the methods used.

5. Randy says the only restriction on the variable x in Marissa and Salvatore's problem is $x \neq 0$. Cynthia says $x \neq 0, -1$. Who is correct? Explain your reasoning.

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6. Calculate each sum or difference. Make sure to list the restrictions for the variable, and simplify when possible.

a. $\frac{5x - 6}{x^2 - 9} - \frac{4}{x - 3}$

b. $\frac{x - 7}{x^2 - 3x + 2} + \frac{4}{x^2 - 7x + 10}$

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c. $\frac{2x - 5}{x} - \frac{4}{5x} - 4$

d. $\frac{3x - 5}{4x^2 + 12x + 9} + \frac{4}{2x + 3} - \frac{2x}{3}$