

Key

Midterm Review

p. 1

Gridded Response

① $x^3 - 1 \div x + 2$

$$x^3 - 1 \Rightarrow |x^3 + 0x^2 + 0x^1 - 1$$

$$x + 2 = 0 \\ \underline{-2} \quad \underline{-2} \\ x = -2$$

$$\begin{array}{r|rrrr} -2 & 1 & 0 & 0 & -1 \\ & \downarrow & -2 & 4 & -8 \\ \hline & 1 & -2 & 4 & -9 \end{array}$$

The remainder is -9.

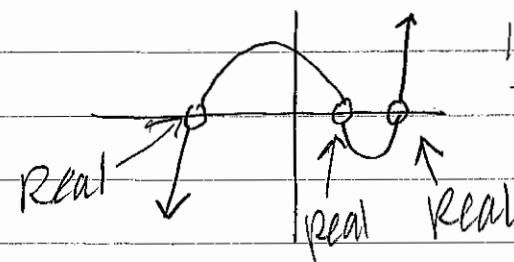
② $f(x) = 2x^5 - 13x^4 + 22x^3 - 187x^2 - 160x + 336$

given factors: $(x-7)$ and $(x+4i)$

other known factor(s): $(x-4i)$ because

imaginary always come in pairs.

So that is 3 so far. The other 2 are both real or both imaginary. The graph



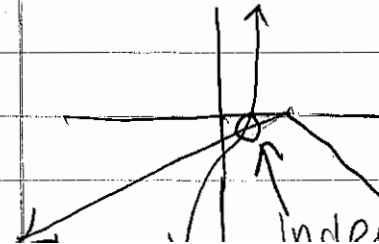
indicates the other two are real

3 Real Zeros
2 Imaginary

③ $f(x) = 2x^3 + 2x - 3$ Find y -value when
 $g(x) = -0.5|x-4|$ $f(x) = g(x)$

In calculator: $y_1 = 2x^3 + 2x - 3$

$$y_2 = -0.5 \text{abs}(x-4)$$



$y = -1.75$

Intersection $(x = -1.75)$

Midterm Review Key p. 2

Gridded Response

$$\textcircled{4} \quad h(x) = \begin{cases} -\frac{1}{2}x - 15 & \text{for } x \leq -4 \\ 20 - 3x^2 & \text{for } x > -4 \end{cases}$$

What is $h(-4) + 3h(-2)$?

$$\begin{aligned} h(-4) &= -\frac{1}{2}(-4) - 15 \\ &= 2 - 15 \\ &= -13 \end{aligned}$$

$$\begin{aligned} h(-2) &= 20 - 3(-2)^2 \\ &= 20 - 3(4) \\ &= 20 - 12 \\ &= 8 \end{aligned}$$
$$\begin{array}{rcc} & h(-4) + 3h(-2) & \\ & \downarrow + \downarrow & \\ & -13 + 24 = \boxed{11} & \end{array}$$

$$3h(-2) = 3(8) = 24$$

$$\textcircled{5} \quad H(x) = 4x^3 - 5x^2 - 23x + 6$$

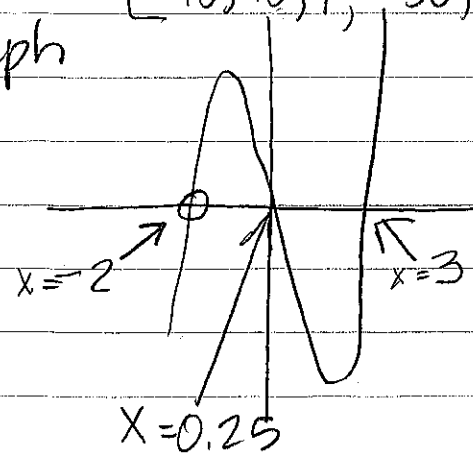
How far apart are the two closest zeros?

In calc:

$$y_1 = 4x^3 - 5x^2 - 23x + 6$$

$[-10, 10, 1, -30, 30, 5]$

Graph



From -2 to 0.25
is $\boxed{2.25 \text{ units}}$

Multiple Choice

⑦ $m(x) = x^3 + 3x^2 - 2x - 4$

one zero is at -1, I can divide to find more.

$$\begin{array}{r|rrrr} -1 & 1 & 3 & -2 & -4 \\ & & \downarrow & -1 & -2 & 4 \\ & 1 & 2 & -4 & 0 \end{array}$$

The result is $x^2 + 2x - 4$. This is not factorable so I know the answers are not pretty.

Quadratic Formula

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$a = 1$

$b = 2$

$c = -4$

$$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-4)}}{2(1)}$$

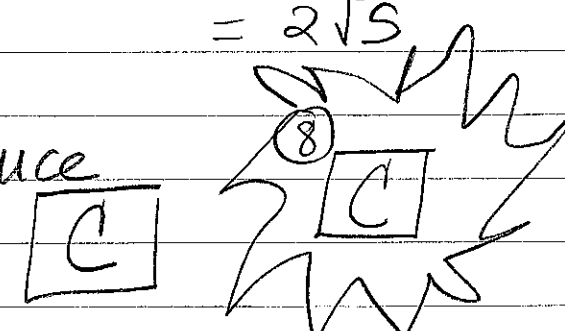
$$x = \frac{-2 \pm \sqrt{4 + 16}}{2}$$

$$\begin{aligned} \sqrt{20} &= \sqrt{4 \cdot 5} \\ &= \sqrt{4} \cdot \sqrt{5} \\ &= 2\sqrt{5} \end{aligned}$$

$$x = \frac{-2 \pm \sqrt{20}}{2}$$

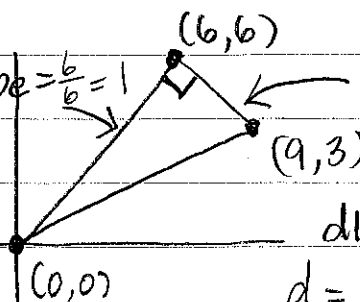
$$x = \frac{-2 \pm 2\sqrt{5}}{2} \text{ Reduce}$$

$$x = -1 \pm \sqrt{5}$$



⑨

Slope = $\frac{6-0}{6-0} = 1$ Slope = $\frac{6-3}{6-9} = \frac{3}{-3} = -1$



These are perpendicular distance from (6,6) to (9,3)

$$\begin{aligned} d &= \sqrt{(9-6)^2 + (3-6)^2} \\ &= \sqrt{(3)^2 + (-3)^2} \\ &= \sqrt{9+9} \end{aligned}$$

⑨ Continued distance from (0,0) to (6,6)

$$d = \sqrt{(0-6)^2 + (0-6)^2}$$

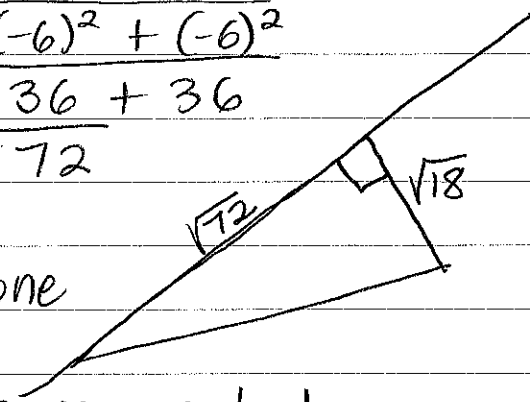
$$d = \sqrt{(-6)^2 + (-6)^2}$$

$$d = \sqrt{36 + 36}$$

$$d = \sqrt{72}$$

$\sqrt{18}$ = radius of cone
Created

$\sqrt{72}$ = height of cone created



$$V = \frac{1}{3} \pi r^2 h$$

$$V = \frac{1}{3} \pi (\sqrt{18})^2 (\sqrt{72})$$

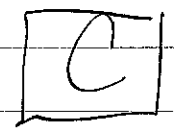
$$V = \frac{1}{3} \pi (18) (\sqrt{72})$$

$$V = 6\pi (\sqrt{72})$$

$$V = 6\pi (6\sqrt{2})$$

$$V = 36\pi \sqrt{2} \text{ units}^3$$

$$\begin{aligned} \sqrt{72} &= \sqrt{36 \cdot 2} \\ &= \sqrt{36} \cdot \sqrt{2} \\ &= 6\sqrt{2} \end{aligned}$$



⑩ $48x^3 - 243xy^2$

$$3x(16x^2 - 81y^2)$$

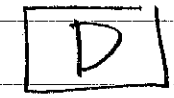
GCF: $3x$

Difference of 2 squares

$$(4x)^2$$

$$(9y)^2$$

$$3x(4x+9y)(4x-9y)$$



⑪ Build factors then multiply

$$x = -1 \rightarrow (x - (-1)) \rightarrow (x + 1) \text{ mult } 2 \rightarrow (x + 1)^2$$

$$x = 2 \rightarrow (x - 2) \quad x = 4 \rightarrow (x - 4)$$

$$p(x) = (x + 1)^2 (x - 2) (x - 4)$$

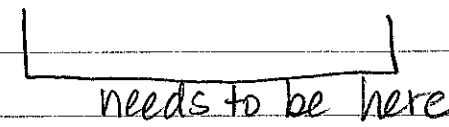
$$p(x) = (x^2 + 2x + 1)(x^2 - 6x + 8)$$

Midterm Review key p.5

① continued $p(x) = x^2(x^2 - 6x + 8) + 2x(x^2 - 6x + 8) + 1(x^2 - 6x + 8)$
 $= x^4 - 6x^3 + 8x^2 + 2x^3 - 12x^2 + 16x + x^2 - 6x + 8$
 $= x^4 - 4x^3 - 3x^2 + 10x + 8$

C

⑫ $31.75 \quad 32.02 \quad 32.25$
 -0.25 +0.25



When in doubt, try them out.

Ⓐ $|x - 0.25| \leq 32$

$$\begin{aligned} -32 &\leq x - 0.25 \leq 32 \\ +0.25 &\quad +0.25 \quad +0.25 \\ -31.75 &\leq x \leq 32.25 \end{aligned}$$

Nope.

Ⓑ $|x + 0.25| \leq 32$

$$-32 \leq x + 0.25 \leq 32$$

Nope. another negative will happen

Ⓒ $|x - 32| \leq 0.25$

$$\begin{aligned} -0.25 &\leq x - 32 \leq 0.25 \\ +32 &\quad +32 \quad +32 \end{aligned}$$

$$31.75 \leq x \leq 32.25 \text{ This one! } \text{☺}$$

C

⑬ $h(x) = \begin{cases} -3x & \text{for } x < 2 \\ 4x + 1 & \text{for } x \geq 2 \end{cases} \quad g(x) = \begin{cases} x^2 + 2 & \text{for } x < 3 \\ x^3 & \text{for } x \geq 3 \end{cases}$

$$3h(2) + 4g(1)$$

$$h(2) = 4(2) + 1 = 8 + 1 = 9$$

$$3h(2) = 3(9) = 27$$

$$g(1) = 1^2 + 2 = 1 + 2 = 3$$

$$4g(1) = 4(3) = 12$$

$$3h(2) + 4g(1)$$

$$\downarrow \quad \downarrow$$

$$27 + 12 = 39$$

A

Midterm Review p.6

(14) $9^{-3x+2} = 48$ variable in the exponent
 base = 9 so logarithm needed
 exp = $-3x+2$
 arg = 48

$b^x = a \rightarrow \log_b a = x$

$\log_9 48 = -3x+2$

change of base

$\frac{\log 48}{\log 9} = -3x+2$

$\log_b a = \frac{\log a}{\log b}$

$\frac{\log 48}{\log 9} - 2 = -3x$

$1.761859507 - 2 = -3x$

$\frac{-0.2381404929}{-3} = x$

$0.07938016 \approx x$

closest .0794



(5) Parent function $y = |x|$

This is shifted down 2 units so the answer is A or B. If not stretched, the slope of absolute value is always 1.

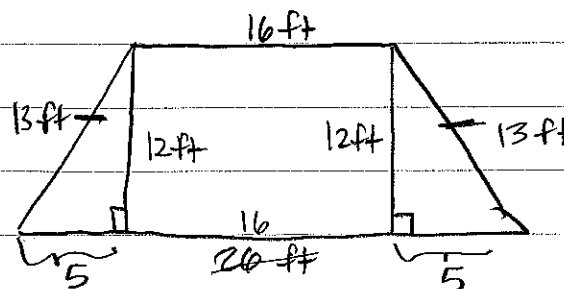
This graph has a slope of $\frac{2}{1}$ or 2. That means



$y = |2x| - 2$

↑ horizontal compression
 ↙ shift down

(16)

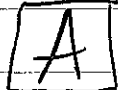


Need to find the area, I made two triangles and a rectangle

$a^2 + b^2 = c^2 \rightarrow b^2 = 144$

$5^2 + b^2 = 13^2 \rightarrow b = 12$

triangles: $2(\frac{1}{2} \cdot 5 \cdot 12) +$
 rectangle: $16 \cdot 12 =$



Midterm Review p.7

$$\textcircled{17} \quad x^2 + y^2 - 4x - 4y + 4 = 0$$

-4 -4 Need to complete
the square

$$(x^2 - 4x) + (y^2 - 4y) = -4$$

$$(x^2 - 4x + \underline{\quad}) + (y^2 - 4y + \underline{\quad}) = -4 + \underline{\quad} + \underline{\quad}$$

$\downarrow \qquad \uparrow \qquad \qquad \uparrow \qquad \qquad \uparrow \quad \uparrow$
 $-\frac{4}{2} = -2 \quad (-2)^2 = 4 \qquad -\frac{4}{2} = -2 \quad (-2)^2 = 4$

$$(x^2 - 4x + 4) + (y^2 - 4y + 4) = -4 + 4 + 4$$

$$(x-2)^2 + (y-2)^2 = 4$$

radius $\sqrt{4} = 2$
center $(2, 2)$

