

Find the center and radius of the circle below. Show all work.

$$3x^2 + 3y^2 - 15x - 30y - 42 = 0$$

1. The coefficient on the squared terms is 3 so I have to divide both sides by 3 before I can complete the square.

$$1. \quad \frac{3}{3}x^2 + \frac{3}{3}y^2 - \frac{15}{3}x - \frac{30}{3}y - \frac{42}{3} = 0$$

$$x^2 + y^2 - 5x - 10y - 14 = 0$$

2. The next thing I needed to do was to move the constant, -14, so I added 14 to both sides.

$$2. \quad x^2 + y^2 - 5x - 10y - 14 + 14 = 0 + 14$$

$$x^2 + y^2 - 5x - 10y = 14$$

3. Then I rearranged my terms so that the x-terms and the y-terms were grouped together.

$$3. \quad x^2 - 5x + y^2 - 10y = 14$$

4. Next I needed to figure out what should be added to the x-terms to make a perfect square trinomial. (I will add the same number on the other side to keep the equation balanced.) I knew that the -5x came from two identical terms being combined so I divided -5 by 2 which gave me $-\frac{5}{2}$ or -2.5. I then squared this to determine what I needed to add.

$$\left(-\frac{5}{2}\right)^2 = \frac{25}{4} \text{ or } 6.25.$$

$$4. \quad (x^2 - 5x + \underline{\quad}) + y^2 - 10y = 14 + \underline{\quad}$$

$$(x^2 - 5x + 6.25) + y^2 - 10y = 14 + 6.25$$

5. After that I completed the square on the y-terms to make a perfect square trinomial. In this case I knew two identical terms were combined to give me -10y. I divided -10 by 2 to get -5. I then squared -5 to determine what I needed to add to both sides. $(-5)^2 = 25$ so I added 25 to both sides of my equation.

$$5. \quad (x^2 - 5x + 6.25) + (y^2 - 10y + \underline{\quad}) = 14 + 6.25 + \underline{\quad}$$

$$(x^2 - 5x + 6.25) + (y^2 - 10y + 25) = 14 + 6.25 + 25$$

6. At this point it was time for me to factor my perfect square trinomials I created and to "clean up" the numbers on the right side of my equation.

$$6. \quad (x - 2.5)^2 + (y - 5)^2 = 45.25$$

7. At last I compared my equation with the standard form equation of a circle:

7. The center is (2.5, 5) and the radius is $\sqrt{45.25} \approx 6.73$.

$$(x - h)^2 + (y - k)^2 = r^2$$

Self-Assessment

| | | |
|---|-----|----|
| I correctly reordered the equation to group the x and y values together. | Yes | No |
| I correctly isolated the constant on the other side of the equation. | Yes | No |
| I understood I needed to complete the square in order to convert the equation to standard form. | Yes | No |
| I correctly completed the square to convert the equation to standard form. | Yes | No |
| I correctly factored the perfect square trinomials. | Yes | No |
| I correctly identified the center from my equation in standard form. | Yes | No |
| I correctly identified the radius from my equation in standard form. | Yes | No |

Additional Practice

1. Complete the square for each of the following:

a) $x^2 + 16x + \underline{\quad}$

c) $x^2 + 7x + \underline{\quad}$

b) $y^2 - 2y + \underline{\quad}$

d) $y^2 + 9y + \underline{\quad}$

2. Factor each as a perfect square:

a) $x^2 - 8x + 16$

b) $x^2 + 18x + 81$

c) $y^2 + 24y + 144$

d) $y^2 - 14y + 49$

e) $x^2 + 5x + 6.25$ or $x^2 + 5x + \frac{25}{4}$

f) $y^2 - 7y + \frac{49}{4}$ or $y^2 - 7y + 12.25$

3. Identify the center and radius of each circle:

a) $(x - 8)^2 + (y - 7)^2 = 81$

Center: Radius: r=

b) $(x + 8)^2 + (y + 7)^2 = 36$

Center: Radius: r=

c) $(x - 8)^2 + (y + 7)^2 = 20$

Center: Radius: r=

d) $\left(x + \frac{1}{2}\right)^2 + \left(y - \frac{7}{4}\right)^2 = \frac{25}{4}$

Center: Radius: r=

4. Now put it all together. Complete the square; write in factored form; and then identify the center and radius of $x^2 + y^2 + 18x - 9y - 19.75$. Show all steps.