

Consider the following expression $\frac{(x+4)(x+2)}{(x-3)(x+2)}$

a) Determine the discontinuities.

I set each factor in the original denominator equal to zero to determine what would cause the expression to be undefined.	$x - 3 = 0$ or $x + 2 = 0$ $x = 3$ $x = -2$ are restrictions of the domain
After I found the restrictions I went back to the expression to determine which, if any, was an asymptote and which was a hole.	$\frac{(x+4)\cancel{(x+2)}}{(x-3)\cancel{(x+2)}} = \frac{(x+4)}{(x-3)}$
Since I was able to reduce the expression by dividing the factor $(x+2)$ into itself, I knew that $x = -2$ was a removable discontinuity, aka a hole. That means the restriction that remained from the factor that I could not reduce (the essential discontinuity) is an actual asymptote: $x=3$.	Hole at $x = -2$ Vertical Asymptote at $x = 3$

b) How do those discontinuities appear on the graph?

Both of the discontinuities are places where the graph cannot occur. The essential discontinuity is the one that did not reduce, so it shows up as an asymptote. That is the invisible boundary line that a graph approaches but cannot cross. The removable discontinuity is the one that did divide out, or reduce, so it shows up as a tiny little hole in the graph. You usually cannot see it so you have to look in the table to confirm.

Additional Practice:

1. Find the values of the removable (hole) and essential (asymptote) discontinuities of $\frac{(x-5)(x+1)}{(x+1)(x+4)}$.

Removable:

Essential:

2. Factor the numerator and denominator of the following expression then find the removable and essential discontinuities.

$$\frac{2x^2 + x - 1}{x^2 - 1}$$

Removable:

Essential:

Consider the following expression $\frac{x^2+6x+8}{x^2-x-6}$

b) Determine the discontinuities.

To find discontinuities of any sort, I knew that I needed to factor the numerator and denominator of the expression.	$\frac{x^2+6x+8}{x^2-x-6} = \frac{(x+4)(x+2)}{(x-3)(x+2)}$
Once I had it factored, I set each factor in the original denominator equal to zero to determine what would cause the expression to be undefined.	$x - 3 = 0$ or $x + 2 = 0$ $x = 3$ $x = -2$ are restrictions of the domain
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