PROBLEM 1 Take Advantage of the Clearance on Expressions

Previously, you learned that adding and subtracting rational expressions involved the same process as adding and subtracting rational numbers. Now, you will see that multiplying rational expressions involves the same steps as multiplying rational numbers.

Remember that when you multiply rational numbers, you can simplify at the beginning or the end, and the product is the same; however, simplifying earlier saves time and will keep the numbers smaller.

1. Consider Method A compared to Method B in both columns of the table.

	Rational Numbers	Rational Expressions Involving Variables
Method A	$\frac{2}{15} \cdot \frac{5}{8} = \frac{10}{120} = \frac{1}{12}$	$\frac{2x}{15x^2} \cdot \frac{5x^2}{8} = \frac{10x^3}{120x^2} = \frac{1x}{12}$
Method B	$\frac{\overset{1}{\cancel{2}}}{\underset{3}{\cancel{15}}} \cdot \frac{\overset{1}{\cancel{5}}}{\underset{4}{\cancel{8}}} = \frac{1}{12}$	$\frac{\overset{1}{2x}}{\underset{3}{\cancel{15x^2}}} \cdot \frac{\overset{1}{\cancel{5x^2}}}{\underset{4}{\cancel{8}}} = \frac{1x}{12}$

a. Explain the difference in the methods.

b. Which method do you prefer?

2. Brody says $x \neq 0$ for the equation in the table, $\frac{2x}{15x^2} \cdot \frac{5x^2}{8} = \frac{1x}{12}$. Damiere says that there are no restrictions because the answer is $\frac{x}{12}$ and there are no variables in the denominator. Who is correct? Explain your reasoning.

3. Analyze Isha's work.

Isha

$$\frac{12xyz^2}{11} \cdot \frac{33x}{8z} = \frac{\overset{9}{\cancel{396}}x^2y\overset{z}{\cancel{z^2}}}{\underset{2}{\cancel{88z}}}$$

$$= \frac{9x^2yz}{2}$$

Explain how Isha could have multiplied the rational expressions more efficiently.

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4. Shaheen multiplies $\frac{5x^2}{3x^2 - 75} \cdot \frac{3x - 15}{4x^2}$ without simplifying first.

Shaheen

$$\frac{5x^2}{3x^2 - 75} \cdot \frac{3x - 15}{4x^2} = \frac{15x^3 - 75x^2}{12x^4 - 300x^2}$$

$$= \frac{15x^2(x - 5)}{3x^2(4x^2 - 100)}$$

$$= \frac{\overset{5}{\cancel{15x^2}}(x - 5)}{\underset{1}{\cancel{3x^2}}(4x^2 - 100)}$$

$$= \frac{5(x - 5)}{4(x^2 - 25)}$$

$$= \frac{5\cancel{(x - 5)}}{4\cancel{(x - 5)}(x + 5)}$$

$$= \frac{5}{4(x + 5)}$$

Complete the same problem as Shaheen, simplifying first, and then list the restrictions.

5. Multiply each expression. List the restrictions for the variables.

a. $\frac{3ab^2}{4c} \cdot \frac{2c^2}{27ab}$

b. $\frac{3x}{5x - 15} \cdot \frac{x - 3}{9x^2}$

c. $\frac{x + 5}{x^2 - 4x + 3} \cdot \frac{x - 3}{4x + 20}$

d. $\frac{7x - 7}{3x^2} \cdot \frac{x + 5}{9x^2 - 9} \cdot \frac{x^2 - 5x - 6}{x^3 + 6x^2 + 5x}$

6. Is the set of rational expressions closed under multiplication? Explain your reasoning.

4. Determine the quotients of each expression.

a. $\frac{9ab^2}{4c} \div \frac{18c^2}{5ab}$

b. $\frac{7x^2}{3x^2 - 27} \div \frac{4x^2}{3x - 9}$

c. $\frac{3x^2 + 15x}{x^2 - 3x - 40} \div \frac{5x^2}{x^2 - 64}$

d. $\frac{4x}{x^2y^2 - xy} \div \frac{x^2 - 4}{3x^2 + 19x - 14} \div \frac{x - 2}{xy}$

5. Is the set of rational expressions closed under division? Explain your reasoning.

Be prepared to share your solutions and methods.