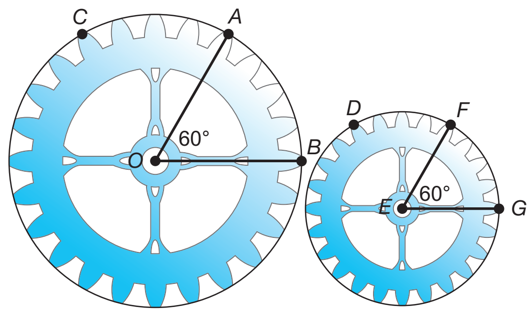
PROBLEM 1 Large and Small Gears

1. Consider the large gear represented by circle O, containing a central angle; $\angle AOB$, whose measure is equal to 60°; a minor arc, $\overset{\frown}{AB}$; and a major arc, $\overset{\frown}{ACB}$, as shown.

   Consider the small gear represented by circle E, containing a central angle; $\angle FEG$, whose measure is equal to 60°; a minor arc, $\overset{\frown}{FG}$; and a major arc, $\overset{\frown}{FDG}$, as shown.



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   a. Is the large gear similar to the small gear? Explain your reasoning.

   b. Is the length of the radii in the large gear proportional to the length of the radii in the small gear? Explain your reasoning.

   c. Determine the degree measure of the minor arc in each circle.

   d. What is the ratio of the degree measure of the minor arc to the degree measure of the entire circle for each of the two gears?

   e. The degree measure of the intercepted arc in the large gear is equal to the degree measure of the intercepted arc in the small gear, but do the two intercepted arcs appear to be the same length?

2. Explain why Casey is incorrect.

Casey

The two minor arcs, $\overset{\frown}{AB}$ and $\overset{\frown}{FG}$, on the gears have the same measure, which is 60°. So, the two arcs are the same length.

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Arc length is a portion of the circumference of a circle. The *length* of an arc is different from the *degree measure* of the arc. Arcs are measured in degrees whereas arc lengths are linear measurements.

To determine the arc length of the minor arc, you need to work with the circumference of the circle, which requires knowing the radius of the circle.

3. If the length of the radius of the large gear, or line segment *OB* is equal to 4 centimeters, determine the circumference of circle *O*.

4. Use the circumference of circle *O* determined in Question 3 and the ratio determined in Question 1, part (d) to solve for the length of the minor arc.

5. If the length of the radius of the small gear, or line segment *EF*, is equal to 2 centimeters, determine the circumference of circle *E*.

6. Use the circumference of circle *E* determined in Question 5 and the ratio determined in Question 1, part (d) to solve for the length of the minor arc.

You determined the arc length by multiplying a fraction that represents the portion of the circumference determined by the central angle and the circumference of the circle. So, the formula for determining arc length, s, can be written as follows.

$$s = \textit{circumference} \cdot \frac{\textit{measure of angle}}{360°}$$

$$s = 2 \cdot \pi \cdot r \cdot \frac{\textit{measure of angle}}{360°}$$

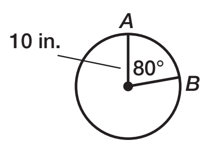
$$s = \frac{\textit{measure of angle}}{360°} \cdot 2\pi r$$

It is important to notice that this formula implies

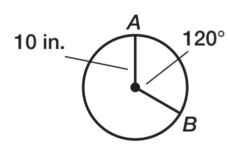
- $\frac{s}{r} = m \cdot \frac{\pi}{180°}$, where m is the measure of the angle.
- $\frac{s}{r}$ is directly proportional to the measure of the central angle, m.

7. Calculate the arc length of each circle. Express your answer in terms of π.

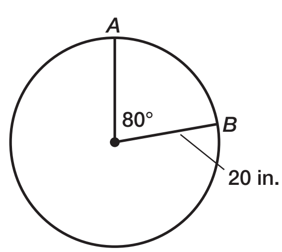
a.



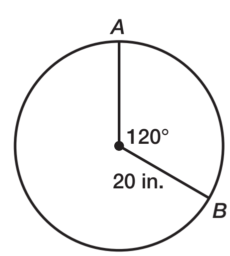
b.



c.

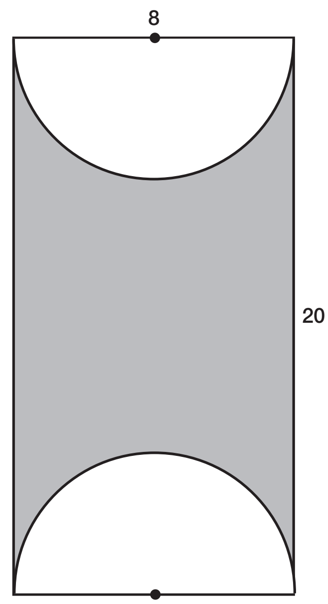


d.



8. Look at Question 7, parts (a) and (c) as well as Question 7, parts (b) and (d). In each pair, the central angle is the same but the radius has been doubled. What effect does doubling the radius have on the length of the arc? Justify why this relationship exist?

9. Two semicircular cuts were taken from the rectangular region shown. Determine the perimeter of the shaded region. Do not express your answer in terms of π.



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