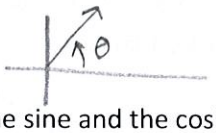
Trigonometric Functions Review Name: Ms. Pace #__

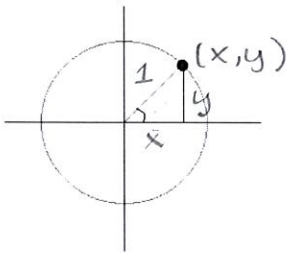
1. In your own words describe what a radian measures. Draw a picture to help with your explanation.

Angle of rotation 

2. What does the independent variable represent for the sine and the cosine function?

independent variable = angle measure

3. Label the picture below and explain how the y-coordinate of the point can be found using the sine function.

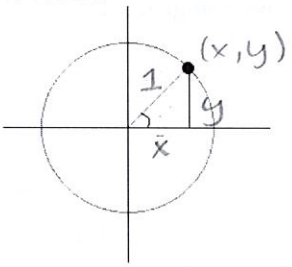


$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$

$\sin\theta = \frac{y}{1}$

$\sin\theta = y$

4. Label the picture below and explain how the x-coordinate of the point can be found using the cosine function.



$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$

$\cos\theta = \frac{x}{1}$

$\cos\theta = x$

5. In words describe how the sine function changes over the interval $0 \le \theta \le 2\pi$

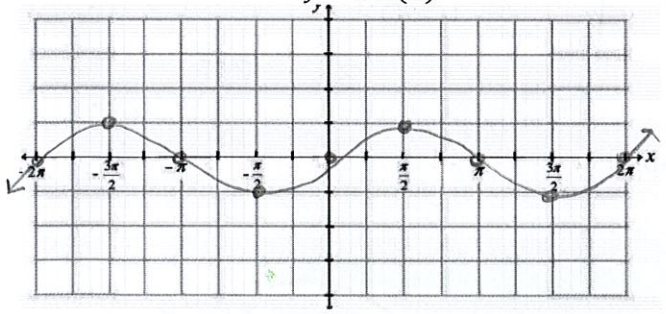
starts @ the midline ($\theta = 0$), increases to maximum ($0, \pi/2$), decreases to minimum ($\pi/2, 3\pi/2$), increases to midline ($3\pi/2, 2\pi$)

6. In words describe how the cosine function changes over the interval $0 \le \theta \le 2\pi$

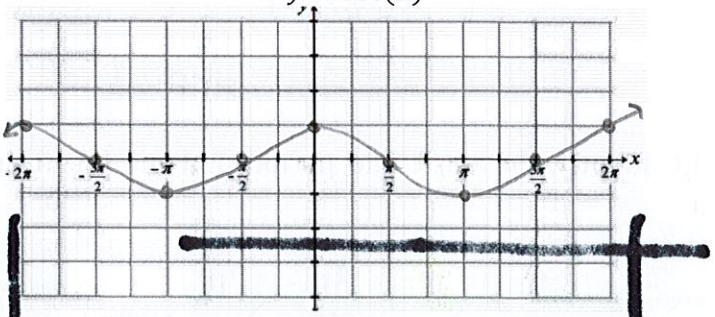
starts @ maximum ($\theta = 0$), decreases to minimum ($0, \pi$), increases to maximum ($\pi, 2\pi$)

7. Without a calculator or your unit circle, create a graph of the sine and cosine function below.

$y = \sin(\theta)$



$y = \cos(\theta)$



8. Convert the following degree measures to radians. Leave in exact form.

a. 20°

$\frac{20}{360} \times \frac{x}{2\pi}$ $x = \frac{\pi}{9}$

$360x = 40\pi$

b. 225°

$\frac{225}{360} \times \frac{x}{2\pi}$ $x = \frac{5\pi}{4}$

$360x = 450\pi$

c. 45°

$\frac{45}{360} \times \frac{x}{2\pi}$ $x = \frac{\pi}{4}$

$360x = 90\pi$

d. -110°

$\frac{-110}{360} \times \frac{x}{2\pi}$ $x = \frac{-11\pi}{18}$

$360x = -220\pi$

9. Convert the following radian measures to degrees.

a. $\frac{\pi}{10}$

$\frac{\pi}{10} \times \frac{x}{360}$

$2\pi x = \frac{360\pi}{10}$

$x = \frac{360\pi}{20\pi}$ $x = 18°$

b. $\frac{7\pi}{15}$

$\frac{7\pi}{15} \times \frac{x}{360}$

$2\pi x = \frac{2520\pi}{15}$

$x = \frac{2520\pi}{30\pi}$ $x = 84°$

c. $\frac{9\pi}{2}$

$\frac{9\pi}{2} \times \frac{x}{360}$

$2\pi x = \frac{3240\pi}{2}$ $x = 810°$

$x = \frac{3240\pi}{4\pi}$

d. $-\frac{3\pi}{2}$

$\frac{-3\pi}{2} \times \frac{x}{360}$

$2\pi x = \frac{-1080\pi}{2}$ $x = -270°$

$x = \frac{-1080\pi}{4\pi}$

radius = 164 feet

10. The largest Ferris wheel in operation is the Cosmolock 21 at Yokohama City, Japan. It has a diameter of 328 feet. Passengers board the cars on a platform to the right halfway up the wheel. The bottom of the wheel is 16.5 feet above ground. Imagine that you have boarded the Comolock 21. The wheel rotates 300° before it stops. How far above the ground are you when your ride on the Cosmolock 21 stops?

$(164(\frac{\sqrt{3}}{2}, -\frac{1}{2})) = (82, -82\sqrt{3})$ ≈ 142 ft. below the midline

midline @ $y = 16.5 + 164 = 180.5$

≈ 38.5 ft

11. A Ferris wheel has a radius of 64 feet. Passengers board the wheel on the right by climbing several flights of stairs to the level of the horizontal axis of the wheel. If a passenger rotates $\frac{5\pi}{6}$ radians before the wheel stops to load another passenger, how high will they be above the ground.

32 ft. above the midline

midline @ $y = 64$ ft.

96 ft

$64(-\frac{\sqrt{3}}{2}, \frac{1}{2})$ → $(-32\sqrt{3}, 32)$ at $\frac{5\pi}{6}$

12. Identify the amplitude, period, and mid-line from an equation.

a. $y = 0.5 \sin\left(\frac{2\pi}{3}x\right) + 1$

amp. = 0.5; period; midline @ $y = 1$

b. $y = 3 \sin\left(\frac{\pi}{2}x\right) + 3$

amp. = 3; midline @ $y = 3$

$\frac{\pi}{2} = \frac{2\pi}{\text{period}}$ period = 4

c. $y = -\cos(2x) - 1$

amp = 1; midline @ $y = -1$

$2 = \frac{2\pi}{\text{period}}$ period = π

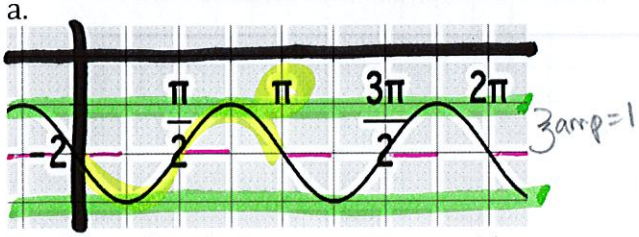
d. $y = 0.5 \sin(3x)$

amp. = 0.5; midline @ $y = 0$

$\frac{3}{1} = \frac{2\pi}{\text{period}}$ $3p = 2\pi$ period = $\frac{2\pi}{3}$

13. Identify the amplitude, period, and mid-line from a graph.

a.



Cosine or Sine? Sine ← starts @ midline

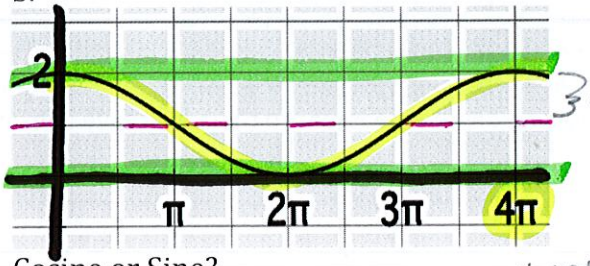
Amplitude: 1

Period: π

Midline: $y = -2$

Equation: $y = -\sin(2x) - 2$ ($\frac{2\pi}{\pi}$; reflection)

b.



Cosine or Sine? Cosine ← starts @ max

Amplitude: 1

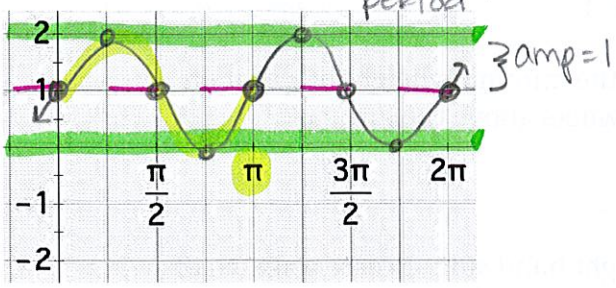
Period: 4π

Midline: $y = 1$

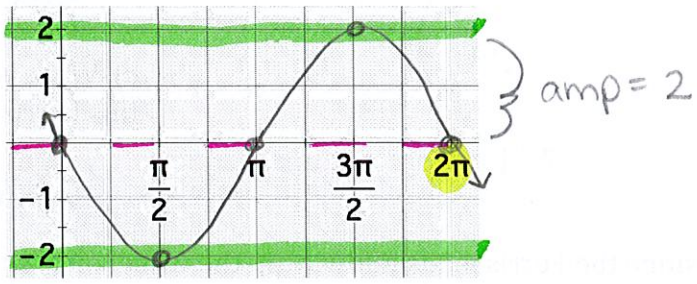
Equation: $y = \cos(\frac{1}{2}x) + 1$ ($\frac{2\pi}{4\pi}$)

14. Given the following equations, use the blank graphs provided to sketch the function.

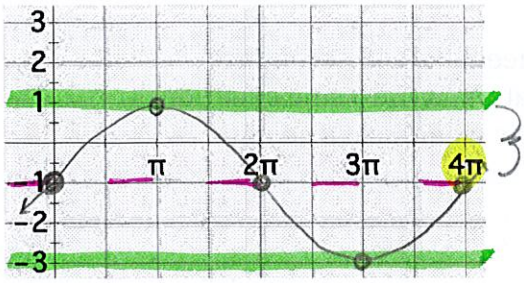
a. $y = \sin(2x) + 1$ ← midline @ $y = 1$; starts @ midline; $2 = \frac{2\pi}{\text{period}}$; period = π



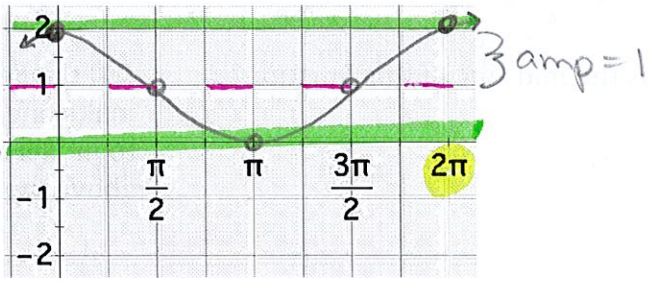
b. $y = -2\sin(x)$ ← midline @ $y = 0$; down first; starts @ midline; period = 2π



c. $y = 2\sin\left(\frac{1}{2}x\right) - 1$ ← midline @ $y = -1$; starts @ midline; $\frac{1}{2} = \frac{2\pi}{\text{period}}$; period = 4π



d. $y = \cos(x) + 1$ ← midline @ $y = 1$; starts @ max; period = 2π



15. Interpret a trig function in context.

The number of hours of daylight in Boston, MA can be modeled by the function with t being days since March 21st :

$$d(t) = 3.5 \sin\left(\frac{2\pi}{365}t\right) + 12.5$$

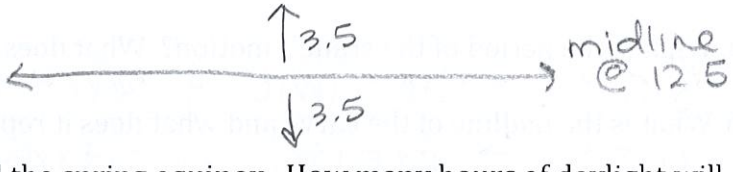
a) Determine the period of this function and explain why it makes sense in the context of the problem.

period = 365 → # of days in a year

b) What is the greatest number of hours of daylight in Boston? What is the least number of hours of daylight in Boston?

max @ 16 hours

min @ 9 hours



c) This function begins on March 21st, this is called the spring equinox. How many hours of daylight will there be on the spring equinox?

12.5 hours → sine functions start @ midline

16. The bottom of a Ferris wheel is located 6 feet from the ground. The wheel has a diameter of 48 feet.

a) Draw a picture of the situation described above.

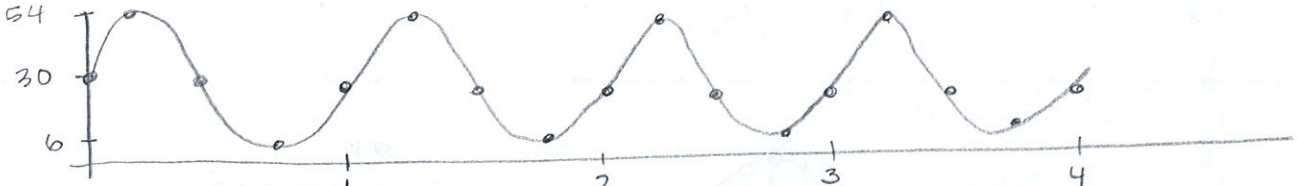
b) What is the maximum height of a seat on this Ferris wheel above the ground?

54ft.

c) What is the minimum height of a seat on this Ferris wheel above the ground?

6ft.

d) Assume the Ferris loads midway up the wheel on the right hand side. Create a sketch of a graph representing the height of the seat above ground over time for 4 revolutions.



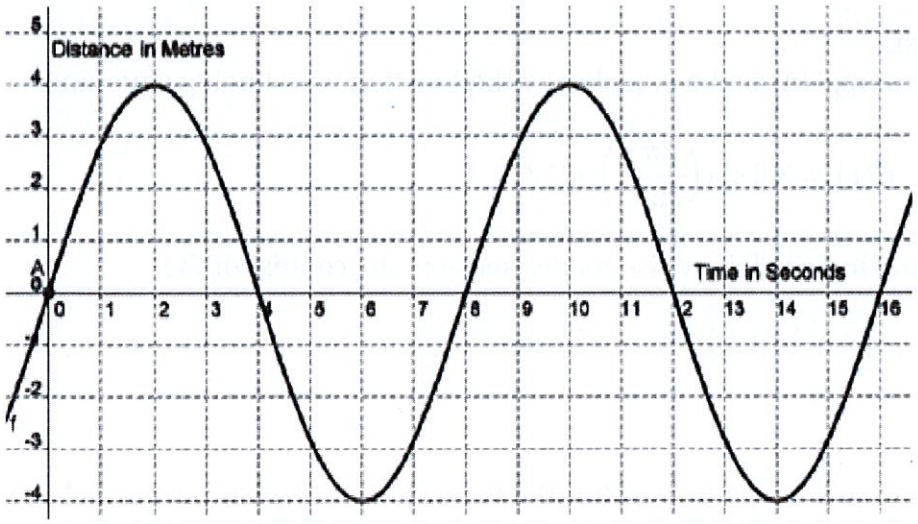
e) What is the mid-line of this function?

$y = 30$

f) If the Ferris Wheel takes 45 seconds to make one full rotation, write a sine equation to model this Ferris wheel seat's height above the ground.

$y = 24\sin\left(\frac{2\pi}{45}x\right) + 30$

17. A wrecking ball attached to a crane swings back and forth. The distance that the ball moves to the left and right of resting position with respect to time is represented by the following graph.



a. What is the period of the crane's motion? What does it represent?

8 sec. → the time it takes to swing back & forth & come back

b. What is the midline of the curve and what does it represent?

$y = 0$ → resting position

c. What is the amplitude of the crane's motion? Draw a diagram to represent what the amplitude represents in terms of motion of the ball.

4 meters

d. Write an equation to model the motion of the wrecking ball.

$y = 4\sin\left(\frac{2\pi}{8}x\right)$