

Below the Line, Above the Line, and Between the Lines

Z-Scores and Percentiles

LEARNING GOALS

In this lesson, you will:

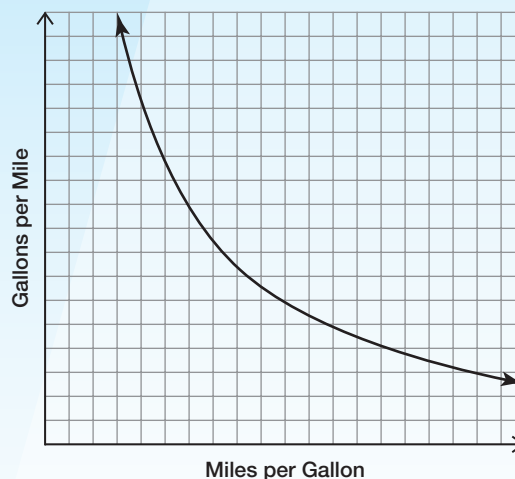
- Use a z-score table to calculate the percent of data below any given data value, above any given data value, and between any two given data values in a normal distribution.
- Use a graphing calculator to calculate the percent of data below any given data value, above any given data value, and between any two given data values in a normal distribution.
- Use a z-score table to determine the data value that represents a given percentile.
- Use a graphing calculator to determine the data value that represents a given percentile.

KEY TERMS

- z-score
- percentile

In 2013, the labels on new vehicles sold in the U.S. got a little different. Instead of just showing how many miles per gallon the vehicle gets, the label also shows the number of gallons per mile it gets.

Why is that important? The reason can be seen on this graph.

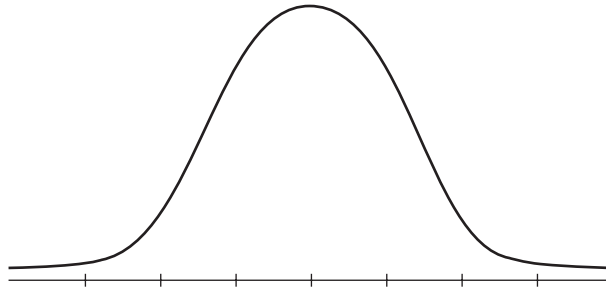


When a car gets a low number of miles per gallon (say, 12 mpg), switching to a slightly higher number (say, 15 mpg) represents a large decrease in the number of gallons per mile, which is a big savings—even bigger than switching from 30 mpg to 50 mpg!

How many gallons per mile does your car get?

PROBLEM 1 Off the Mark

1. The fuel efficiency of a sample of hybrid cars is normally distributed with a mean of 54 miles per gallon (mpg) and a standard deviation of 6 miles per gallon.
- a. Use the mean and standard deviation to label the intervals on the horizontal axis of the normal curve in miles per gallon.



- b. Determine the percent of hybrid cars that get less than 60 miles per gallon. Explain your reasoning.
- c. Determine the percent of hybrid cars that get less than 66 miles per gallon. Explain your reasoning.
- d. Determine the percent of hybrid cars that get less than 72 miles per gallon. Explain your reasoning.

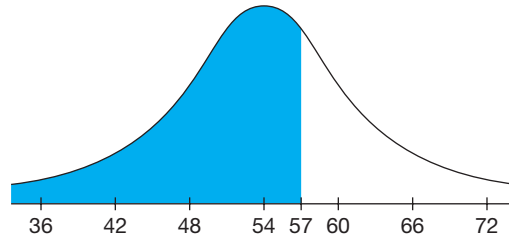




When data values are aligned with integer multiples of the standard deviation from the mean, you can use the Empirical Rule for Normal Distributions to calculate the percent of data values less than that value. But what if a data value does not align with the standard deviations?




2. Let's consider the fuel efficiency of hybrid cars again. The mean is 54 miles per gallon and 1 standard deviation is 6 miles per gallon. What percent of cars get less than 57 miles per gallon?



- a. How many standard deviations from the mean is 57 miles per gallon? Explain how you determined your answer.



- b. Greg incorrectly estimated the percent of hybrid cars that get less than 57 miles per gallon.

 **Greg**

Approximately 67% of hybrid cars get less than 57 miles per gallon.

$$50\% + \frac{1}{2}(34\%) \approx 67\%$$

Explain why Greg's reasoning is incorrect.



The number you calculated in Question 2 is a *z-score*. A **z-score** is a number that describes a specific data value's distance from the mean in terms of standard deviation units.

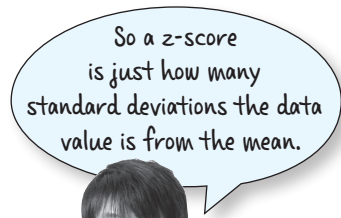
For a population, a z-score is determined by the equation

$$z = \frac{(x - \mu)}{\sigma},$$

where x represents a value from the data.

You can use a z-score table to determine the percent of data less than a given data value with a corresponding z-score.

A z-score table is provided at the end of this lesson.



To determine the percent of hybrid cars that get less than 57 miles per gallon with a z-score table, first calculate the z-score for 57 miles per gallon. In Question 2, you calculated the score for 57 miles per gallon as 0.5.

z	0.0	0.01	0.02	0.03	0.04	0.05
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199
0.1	0.5398	0.5433	0.5478	0.5517	0.5557	0.5596
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5987
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7088

Next, locate the row that represents the ones and tenths place of the z-score. For a z-score of 0.5, this is the row labeled 0.5. Also locate the column that represents the hundredths place of the z-score. For a z-score of 0.5, this is the column labeled 0.0. Note that the table represents z-scores only to the hundredths place.

Finally, locate the cell that is the intersection of the row and column. The numbers in each cell represent the percent of data values below each z-score. For a z-score of 0.5, the corresponding cell reads 0.6915.

This means that 69.15% of hybrid cars get less than 57 miles per gallon.



3. What would a negative z-score indicate? Explain your reasoning.

A graphing calculator can determine a percent of data below a z-score. This function is called the normal cumulative density function (normalcdf). The function determines the percent of data values within an interval of a normal distribution.



You can also use a graphing calculator to determine the percent of data below a given z-score.

Step 1: Press **2nd** and then **VARΣ**.
Select **2:normalcdf(**

Step 2: Enter the lower bound of the interval, the upper bound of the interval, the mean, and the standard deviation, including commas between each number.

Step 3: Press **ENTER**. Calculator results will vary from results obtained using a z-score table.

In this problem, the lower bound is negative infinity, the upper bound is 57, the mean is 54, and the standard deviation is 6. Most calculators do not have an infinity button. You can use -1×10^{99} , a very small number, for the lower bound.

4. Use a graphing calculator to determine the approximate percent of hybrid cars that get less than 57 miles per gallon.



5. How does this answer compare to the answer you got by using the z-score table?



6. Juan and Michael used a graphing calculator to determine that the percent of hybrid cars that get less than 57 miles per gallon is approximately 69.15%. Juan entered values in standard deviation units, and Michael entered values in terms of miles per gallon.



Juan

```
normalcdf(-9,0.5,  
0,1)  
.6914624678
```



Michael

```
normalcdf(0,57,  
54,6)  
.6914624678
```

- a. Explain why Juan and Carlos used different values but still got the same result.
- b. Explain why Michael used 0 for the lower bound of the interval. Explain why Juan used -9 for the lower bound of the interval.



7. Josh calculated the percent of hybrid cars that get less than 56 miles per gallon using the z-score table and a graphing calculator.

z	0.0	0.01	0.02	0.03
0.0	0.5000	0.5040	0.5080	0.5120
0.1	0.5398	0.5438	0.5478	0.5517
0.2	0.5793	0.5832	0.5871	0.5910
0.3	0.6179	0.6217	0.6255	0.6293

```
Josh  
normalcdf(-1E99,  
56,54,6)  
.6305585963
```

Explain why Josh received different results from the z-score table and the graphing calculator.

PROBLEM 2 More or Less . . .



1. Calculate the percent of hybrid cars that get less than 50 miles per gallon.

2. Use your answer to Question 1 to calculate the percent of hybrid cars that get more than 50 miles per gallon. Explain your reasoning.

3. Calculate the percent of hybrid cars that get less than 50 miles per gallon and the percent of hybrid cars that get less than 60 miles per gallon.



4. Use your answer to Question 3 to calculate the percent of hybrid cars that get between 50 and 60 miles per gallon. Explain your reasoning.

PROBLEM 3 Top Texters



You may have heard someone say, “My baby’s weight is in the 90th percentile” or, “My student scored in the 80th percentile in math.” What do these phrases mean?

A **percentile** is a data value for which a certain percentage of the data is below the data value in a normal distribution. For example, 90% of the data in a set is below the value at the 90th percentile, and 80% of the data is below the value at the 80th percentile.

The number of text messages teens send and receive every day can be represented as a normal distribution with a mean of 100 text messages per day and a standard deviation of 25 texts per day.



1. Calculate the 50th percentile for this data set. Explain your reasoning.
2. Would a teen in the 90th percentile send and receive more or fewer than 100 text messages per day? Explain your reasoning.
3. Would a teen in the 10th percentile send and receive more or fewer than 100 text messages per day? Explain your reasoning.

4. Use a z-score table to determine the 90th percentile for teen text messages.
- Determine the percent value in the z-score table that is closest to 90%. Explain what information the z-score provides.
 - Calculate the 90th percentile. Show your work.



You can also use a graphing calculator to calculate a percentile. To calculate a percentile, you can use the inverse of the normal cumulative density function (invNorm). The invNorm function takes a percent as input and returns the data value.



You can use a graphing calculator to determine the total number of text messages that correspond to the 90th percentile.

Step 1: Press **2nd** and then **VARΣ**.
Select **3:invNorm(**

Step 2: Enter the percentile in decimal form, the mean, and the standard deviation, including commas between each number.

Step 3: Press **ENTER**.

In the texting situation, the percentile is 0.90, the mean is 100, and the standard deviation is 25.

5. Determine the total number of text messages that represent the 20th percentile.



Be prepared to share your solutions and methods.

z-scores and percent of data below the z-score										
z	0.09	0.08	0.07	0.06	0.05	0.04	0.03	0.02	0.01	0.0
-3.4	0.0002	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003	0.0003
-3.3	0.0003	0.0004	0.0004	0.0004	0.0004	0.0004	0.0004	0.0005	0.0005	0.0005
-3.2	0.0005	0.0005	0.0005	0.0006	0.0006	0.0006	0.0006	0.0006	0.0007	0.0007
-3.1	0.0007	0.0007	0.0008	0.0008	0.0008	0.0008	0.0009	0.0009	0.0009	0.0010
-3.0	0.0010	0.0010	0.0011	0.0011	0.0011	0.0012	0.0012	0.0013	0.0013	0.0013
-2.9	0.0014	0.0014	0.0015	0.0015	0.0016	0.0016	0.0017	0.0018	0.0018	0.0019
-2.8	0.0019	0.0020	0.0021	0.0021	0.0022	0.0023	0.0023	0.0024	0.0025	0.0026
-2.7	0.0026	0.0027	0.0028	0.0029	0.0030	0.0031	0.0032	0.0033	0.0034	0.0035
-2.6	0.0036	0.0037	0.0038	0.0039	0.0040	0.0041	0.0043	0.0044	0.0045	0.0047
-2.5	0.0048	0.0049	0.0051	0.0052	0.0054	0.0055	0.0057	0.0059	0.0060	0.0062
-2.4	0.0064	0.0066	0.0068	0.0069	0.0071	0.0073	0.0075	0.0078	0.0080	0.0082
-2.3	0.0084	0.0087	0.0089	0.0091	0.0094	0.0096	0.0099	0.0102	0.0104	0.0107
-2.2	0.0110	0.0113	0.0116	0.0119	0.0122	0.0125	0.0129	0.0132	0.0136	0.0139
-2.1	0.0143	0.0146	0.0150	0.0154	0.0158	0.0162	0.0166	0.0170	0.0174	0.0179
-2.0	0.0183	0.0188	0.0192	0.0197	0.0202	0.0207	0.0212	0.0217	0.0222	0.0228
-1.9	0.0233	0.0239	0.0244	0.0250	0.0256	0.0262	0.0268	0.0274	0.0281	0.0287
-1.8	0.0294	0.0301	0.0307	0.0314	0.0322	0.0329	0.0336	0.0344	0.0351	0.0359
-1.7	0.0367	0.0375	0.0384	0.0392	0.0401	0.0409	0.0418	0.0427	0.0436	0.0446
-1.6	0.0455	0.0465	0.0475	0.0485	0.0495	0.0505	0.0516	0.0526	0.0537	0.0548
-1.5	0.0559	0.0571	0.0582	0.0594	0.0606	0.0618	0.0630	0.0643	0.0655	0.0668
-1.4	0.0681	0.0694	0.0708	0.0721	0.0735	0.0749	0.0764	0.0778	0.0793	0.0808
-1.3	0.0823	0.0838	0.0853	0.0869	0.0885	0.0901	0.0918	0.0934	0.0951	0.0968
-1.2	0.0985	0.1003	0.1020	0.1038	0.1056	0.1075	0.1093	0.1112	0.1131	0.1151
-1.1	0.1170	0.1190	0.1210	0.1230	0.1251	0.1271	0.1292	0.1314	0.1335	0.1357
-1.0	0.1379	0.1401	0.1423	0.1446	0.1469	0.1492	0.1515	0.1539	0.1562	0.1587
-0.9	0.1611	0.1635	0.1660	0.1685	0.1711	0.1736	0.1762	0.1788	0.1814	0.1841
-0.8	0.1867	0.1894	0.1922	0.1949	0.1977	0.2005	0.2033	0.2061	0.2090	0.2119
-0.7	0.2148	0.2177	0.2206	0.2236	0.2266	0.2296	0.2327	0.2358	0.2389	0.2420
-0.6	0.2451	0.2483	0.2514	0.2546	0.2578	0.2611	0.2643	0.2676	0.2709	0.2743
-0.5	0.2776	0.2810	0.2843	0.2877	0.2912	0.2946	0.2981	0.3015	0.3050	0.3085
-0.4	0.3121	0.3156	0.3192	0.3228	0.3264	0.3300	0.3336	0.3372	0.3409	0.3446
-0.3	0.3483	0.3520	0.3557	0.3594	0.3632	0.3669	0.3707	0.3745	0.3783	0.3821
-0.2	0.3829	0.3897	0.3936	0.3974	0.4013	0.4052	0.4090	0.4129	0.4168	0.4207
-0.1	0.4247	0.4286	0.4325	0.4364	0.4404	0.4443	0.4483	0.4522	0.4562	0.4602
-0.0	0.4641	0.4681	0.4721	0.4761	0.4801	0.4840	0.4880	0.4920	0.4960	0.5000

z	0.0	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.5000	0.5040	0.5080	0.5120	0.5160	0.5199	0.5239	0.5279	0.5319	0.5359
0.1	0.5398	0.5438	0.5478	0.5517	0.5557	0.5596	0.5636	0.5875	0.5714	0.5753
0.2	0.5793	0.5832	0.5871	0.5910	0.5948	0.5967	0.6026	0.6064	0.6103	0.6141
0.3	0.6179	0.6217	0.6255	0.6293	0.6331	0.6368	0.6406	0.6443	0.6480	0.6517
0.4	0.6554	0.6591	0.6628	0.6664	0.6700	0.6736	0.6772	0.6808	0.6844	0.6879
0.5	0.6915	0.6950	0.6985	0.7019	0.7054	0.7068	0.7123	0.7157	0.7190	0.7224
0.6	0.7257	0.7291	0.7324	0.7357	0.7389	0.7422	0.7454	0.7486	0.7517	0.7549
0.7	0.7580	0.7611	0.7642	0.7673	0.7704	0.7734	0.7764	0.7794	0.7823	0.7852
0.8	0.7881	0.7910	0.7939	0.7967	0.7995	0.8023	0.8051	0.8078	0.8106	0.8133
0.9	0.8159	0.8186	0.8212	0.8238	0.8264	0.8269	0.8315	0.8340	0.8365	0.8389
1.0	0.8413	0.8438	0.8461	0.8485	0.8508	0.8531	0.8554	0.8577	0.8599	0.8621
1.1	0.8643	0.8665	0.8686	0.8708	0.8729	0.8749	0.8770	0.8790	0.8810	0.8830
1.2	0.8849	0.8869	0.8888	0.8907	0.8925	0.8944	0.8962	0.8960	0.8997	0.9015
1.3	0.9032	0.9049	0.9066	0.9082	0.9099	0.9115	0.9131	0.9147	0.9162	0.9177
1.4	0.9192	0.9207	0.9222	0.9236	0.9251	0.9265	0.9279	0.9292	0.9306	0.9319
1.5	0.9332	0.9345	0.9357	0.9370	0.9382	0.9394	0.9406	0.9418	0.9429	0.9441
1.6	0.9452	0.9463	0.9474	0.9484	0.9495	0.9505	0.9515	0.9525	0.9535	0.9545
1.7	0.9554	0.9564	0.9573	0.9582	0.9591	0.9599	0.9608	0.9616	0.9625	0.9633
1.8	0.9641	0.9649	0.9656	0.9664	0.9671	0.9678	0.9666	0.9693	0.9699	0.9706
1.9	0.9713	0.9719	0.9726	0.9732	0.9738	0.9744	0.9750	0.9756	0.9761	0.9767
2.0	0.9772	0.9778	0.9783	0.9788	0.9793	0.9798	0.9803	0.9808	0.9812	0.9817
2.1	0.9821	0.9826	0.9830	0.9834	0.9838	0.9842	0.9846	0.9850	0.9854	0.9857
2.2	0.9861	0.9864	0.9868	0.9871	0.9875	0.9878	0.9881	0.9884	0.9887	0.9890
2.3	0.9893	0.9896	0.9898	0.9901	0.9904	0.9906	0.9909	0.9911	0.9913	0.9916
2.4	0.9918	0.9920	0.9922	0.9925	0.9927	0.9929	0.9931	0.9932	0.9934	0.9936
2.5	0.9938	0.9940	0.9941	0.9943	0.9945	0.9946	0.9948	0.9949	0.9951	0.9952
2.6	0.9953	0.9955	0.9956	0.9957	0.9959	0.9960	0.9961	0.9962	0.9963	0.9964
2.7	0.9965	0.9966	0.9967	0.9968	0.9969	0.9970	0.9971	0.9972	0.9973	0.9974
2.8	0.9974	0.9975	0.9976	0.9977	0.9977	0.9978	0.9979	0.9979	0.9980	0.9981
2.9	0.9981	0.9982	0.9982	0.9983	0.9984	0.9984	0.9985	0.9985	0.9986	0.9986
3.0	0.9987	0.9987	0.9987	0.9988	0.9988	0.9989	0.9989	0.9989	0.9990	0.9990
3.1	0.9990	0.9991	0.9991	0.9991	0.9992	0.9992	0.9992	0.9992	0.9993	0.9993
3.2	0.9993	0.9993	0.9994	0.9994	0.9994	0.9994	0.9994	0.9995	0.9995	0.9995
3.3	0.9995	0.9995	0.9995	0.9996	0.9996	0.9996	0.9996	0.9996	0.9996	0.9997
3.4	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9997	0.9998

You Make the Call

Normal Distributions and Probability

LEARNING GOALS

In this lesson, you will:

- Interpret a normal curve in terms of probability.
- Use normal distributions to determine probabilities.
- Use normal distributions and probabilities to make decisions.

You can grow both tomatoes and potatoes easily in a home garden—separately—but what about a plant that can grow both vegetables (or is it a fruit and a vegetable?) at the same time?

A company based in the United Kingdom created just that—a hybrid plant that produces both tomatoes above the ground and potatoes below. This remarkable plant was not created through genetic engineering, but rather by grafting the two types of plants together at the stem.

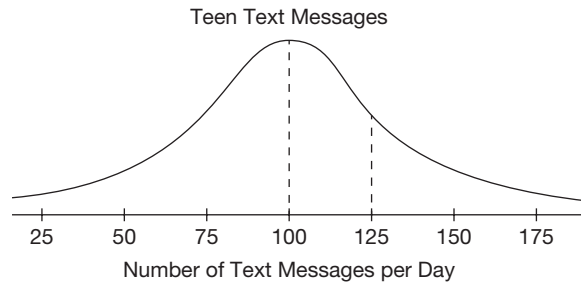
Now for the important question: What would you call this hybrid plant?

PROBLEM 1 Teens and Texting

So far, you have explored the percent of data values that fall within specified intervals. However, you can also interpret a normal distribution in terms of probabilities.



Based on a survey, the number of text messages that teens send and receive every day is a normal distribution with a mean of 100 text messages per day and a standard deviation of 25 text messages per day.



You randomly select a teen from the survey. Calculate each probability.

1. Determine the probability that the randomly selected teen sends and receives between 100 and 125 text messages per day.
2. Determine the probability that the randomly selected teen sends and receives fewer than 75 text messages per day.



3. Determine the probability that the randomly selected teen sends and receives more than 140 text messages per day.

PROBLEM 2 Pizza Anyone?

1



You have collected data on the delivery times for two local pizza shops, Antonio's Pizza and Wood Fire Pizza. Based on your data, Antonio's Pizza has a mean delivery time of 30 minutes and a standard deviation of 3 minutes. Wood Fired Pizza has a mean delivery time of 25 minutes and a standard deviation of 8 minutes.

1. What factors could influence the delivery time of an order from either pizza shop?

2. What can you conclude based only on the mean and standard deviation for each pizza shop?



3. A friend of yours is planning a party. She needs the pizza for the party delivered in 35 minutes or less or the party will be a complete disaster! Which pizza shop has a greater probability of delivering the order within 35 minutes?

PROBLEM 3 You Say Tomato, I Say Prize-Winning Tomato!



1. Brad and Toby both plan to enter the county tomato growing competition. Each person who enters the competition must submit a basket of tomatoes. The judges randomly select a tomato from each contestant's basket. According to the rules of the competition, a "golden" tomato has a diameter between 4 inches and 4.5 inches.
The diameters of tomatoes in Brad's basket are normally distributed with a mean diameter of 3.6 inches and a standard deviation of 1 inch. The diameters of tomatoes in Toby's basket are also normally distributed with a mean diameter of 3.8 inches and a standard deviation of 0.2 inches.
When the judges randomly select a tomato from Brad's and Toby's basket, whose is more likely to result in a "golden" tomato?



Be prepared to share your solutions and methods.